



**THORPE**  
PRIMARY SCHOOL

**Calculation Policy 2023**

# EYFS – Year 6 Mathematics

## The Importance of Mental Mathematics

While this policy focuses on written calculation in mathematics, we recognise the importance of mental strategies and known facts that form the basis of all calculations. Pupils are provided with frequent opportunities to compare and evaluate different calculation strategies. This helps them develop an understanding that efficiency is personal and based on the numbers involved. Mental maths fluency underpins all effective written calculation approaches.

## Concrete, Pictorial and Abstract

**Concrete** - Manipulatives are objects that can be touched and moved by pupils to introduce, explore or reinforce a mathematical concept. They provide a vehicle to help pupils make sense of complex, symbolic and abstract ideas through exploration and manipulation. Furthermore, they support the development of internal models and help build stronger memory pathways. All pupils should have frequent opportunities to develop their understanding of mathematical concepts through the appropriate use of concrete apparatus.

Concrete resources that may be found in classrooms will include:



These resources will vary depending on year group and individual needs. At home, pupils very well may not have access to these school resources; however, they are just a vehicle to support a pupil's understanding of a topic. Any objects can be used at home to replace counters, cubes etc.

**Pictorial (including jottings)** - The act of translating the concrete experience into a pictorial representation helps focus attention on what has happened and why. This supports deeper understanding and a stronger imprint on memory. Pictorial representations are more malleable than concrete resources and, once understanding is secured, allow exploration of complex problems that may be challenging to reproduce with manipulatives. When a child is working at the pictorial stage, it often provides rich opportunities for assessment of their depth of understanding.

**Abstract - Written** The aim, within this policy, is for compacted forms of notation. These have developed through the history of mathematics. Explicit individual steps in procedure are hidden or they have been shortcut. The informal and expanded methods expose all the intermediate steps, replicating thought processes more closely and support understanding prior to compaction.

**Abstract - Spoken Learning** to use the correct mathematical vocabulary is vital for the development of mathematical proficiency. The ability to articulate accurately allows pupils to communicate and build meaning. Ideas become more permanent. This can be scaffolded effectively using speaking frames.

### Impact

Pupils will leave us prepared for the next stage in their lives with:

- Pupils have an appreciation for the maths in everyday life
- Quick recall of facts and procedures
- The flexibility and fluidity to move between different contexts and representations of mathematics
- The ability to recognise relationships and make connections in mathematics
- Confidence and belief that they can achieve
- The knowledge that maths underpins most of our daily lives
- Skills and concepts that have been mastered
- Have a positive and inquisitive attitude to mathematics as an interesting and attractive subject in which all children gain success and pleasure.



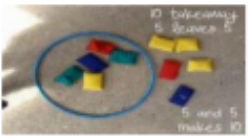

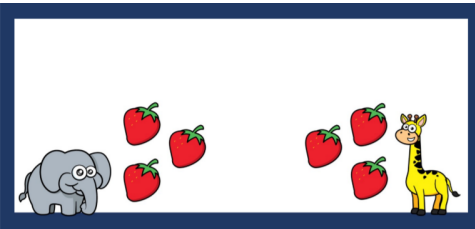
A mathematical concept or skill has been mastered when a child can show it in multiple ways, using the mathematical language to

explain their ideas, and can independently apply the concept to new problems in unfamiliar situations and this is the goal for our children. These will be assessed through: assessment, tracking, pupil progress meetings, performance management, moderation and standardisation.

### Early Years Foundation Stage

Children at the expected level of development will:

- Have a deep understanding of number to 10, including the composition of each number
- Subitise (recognise quantities without counting) up to 5
- Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Addition	Subtraction	Multiplication	Division
<p>Children are encouraged to gain a sense of the number system through the use of counting concrete objects.</p>  <p>7 ways and count all.</p>  <p>They understand addition as</p>	<p>Children are encouraged to gain a sense of the number system through the use of counting concrete objects and understand subtraction as counting out.</p>  <p>They begin to count back in ones and twos using objects, cubes, bead string and number line.</p>	<p>Children use concrete objects to make and count equal groups of objects.</p>  <p>They will count on in twos using a bead string and number line.</p> <p>They understand doubling as repeated addition.</p>	<p>Children use concrete objects to count and share equally into 2 groups.</p>  <p>They count a set of objects and halve them by making two equal groups.</p>

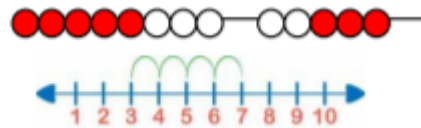


counting on and will count on in ones and twos using objects, cubes, bead string and number line.



They use concrete and pictorial representation to record their calculations.

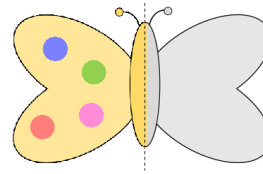
When confident children may be able to pictorially represent their calculations using symbols and numbers within a written



They may use concrete and pictorial representation to record their calculations.

They are encouraged to develop a mental picture of the number system in their heads to use for calculations. When confident children may be able to represent their calculations using symbols and numbers within a written calculation.

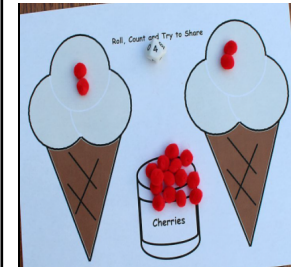
$$4+4=8$$



They use concrete and pictorial representation to record their calculations. When confident children may be able to represent their calculations using symbols and numbers within a written calculation.



They understand sharing and halving as dividing by 2.



They will begin to use objects to make groups of 2 from a given amount. They use concrete and pictorial representation to record their calculations.

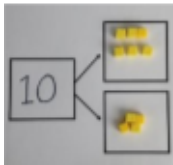


## Year 1 Addition

Pupils should be able to:

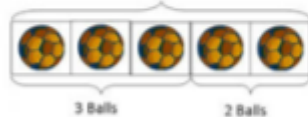
- read, write and interpret mathematical statements involving addition (+) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as  $7 = \square + 9$ .

Combining two parts to make a whole: part- whole model

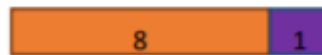


Use part part whole model.


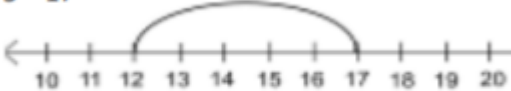



Use cubes to add two numbers together as a group or in a bar.



Use pictures to add two numbers together as a group or in a bar.



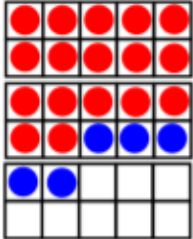
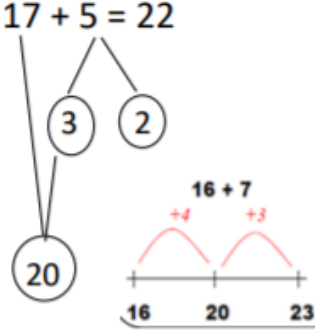
Part whole model drawing


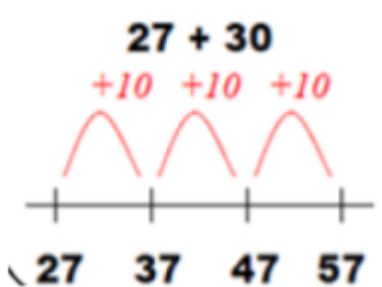
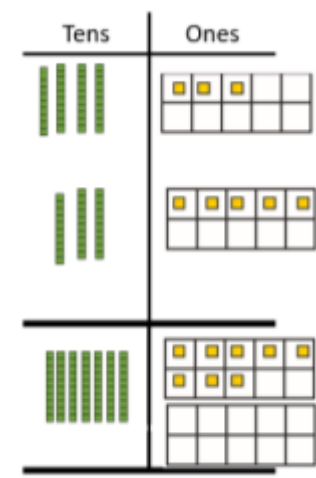
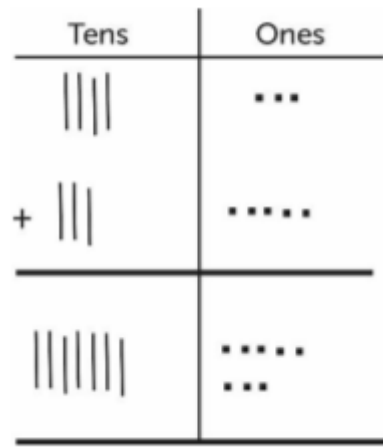
<p>Starting with the biggest number and counting on</p>	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	<p><math>12 + 5 = 17</math></p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>Place the larger number in your head and count on the smaller number to find your answer.</p>
<p>Regrouping to make 10. (This is an essential skill for column addition later).</p>	 <p><math>6 + 5 = 11</math></p>  <p>Start with the bigger number and use the smaller number to make 10. Use ten frames.</p>	 <p><math>3 + 9 =</math></p> <p>Use pictures or a number line. Regroup or partition the smaller number using the part whole model to make 10.</p>	<p><math>7 + 4 = 11</math></p> <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p>

## Year 2 Addition

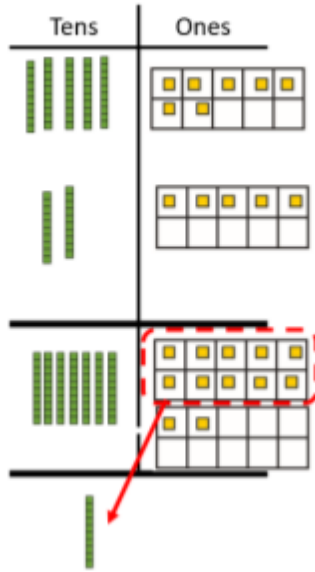
Pupils should be taught to:

- add numbers using concrete objects, pictorial representations, and mentally including:
  - a two-digit number and ones a two-digit number
  - tens two two-digit numbers
  - adding three one-digit numbers
- solve problems with addition: using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- recall and use addition facts to 20 fluently, and derive and use related facts up to 100

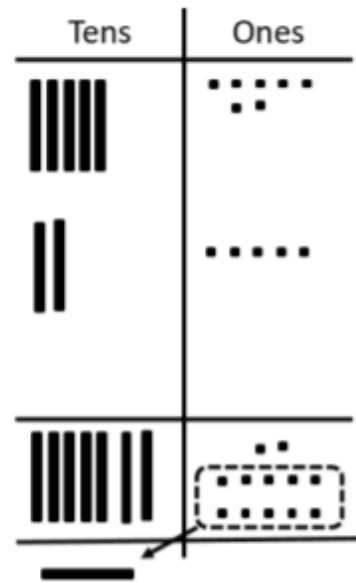
Objectives and strategy	Concrete	Pictorial	Abstract
<p>Add a two digit number and ones</p>	 <p style="text-align: center;"><math>17 + 5 = 22</math></p> <p style="text-align: center;">Use ten frame to make 'magic ten</p> <p>Children explore the pattern.</p> <p><math>17 + 5 = 22</math></p> <p><math>27 + 5 = 32</math></p>	<p>Use part part whole and number line to model.</p> <div style="text-align: center;"> <p><math>17 + 5 = 22</math></p>  </div>	

<p>Add a 2 digit number and tens</p>	 <p><math>25 + 10 = 35</math></p> <p>Explore that the ones digit does not change</p>	 <p><math>27 + 30</math></p> <p><math>+10 +10 +10</math></p> <p>27 37 47 57</p>	<p><math>27 + 10 = 37</math></p> <p><math>27 + 20 = 47</math></p> <p><math>27 + \square = 57</math></p>
<p>Add two 2-digit numbers (without regrouping)</p>	 <p>Using base-ten, numicon and place value counters to support adding two 2-digit numbers without regrouping.</p>	 <p>When confident they will begin to draw the expanded written method.</p> <p>We provide lots of strategies for adding two digit numbers for example number lines -</p>	<p>Adding tens and ones -</p> $\begin{array}{r} 40 \ 3 \\ + 30 \ 5 \\ \hline 70 \ 8 \end{array}$ <p><math>43 + 35 = 78</math></p> <p>When confident they will begin to use the expanded written method to add two digit numbers.</p>

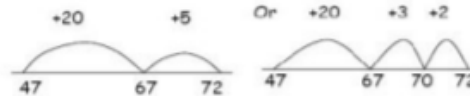
Add two digit numbers with regrouping (through ten)



Add the ones and regroup them on the tens frame to exchange for a ten. Add the tens together.



After practically using the base ten, they will begin to draw using a pictorial representation of the expanded written method (with regrouping).



Use number line and bridge ten using part whole if necessary.

When they are confident they will begin to use the expanded written method to add two two -digit numbers with regrouping.

$$\begin{array}{r}
 50 + 7 \\
 + 20 + 5 \\
 \hline
 70 + 12 \\
 \hline
 10 \quad 2 \\
 \hline
 10
 \end{array}$$

$57 + 25 = 82$

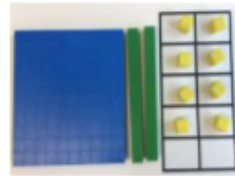
Children will also have the opportunity to use partitioning for addition.

$$\begin{array}{l}
 25 + 47 \\
 \swarrow \quad \searrow \\
 20 + 5 \quad 40 + 7 \\
 20 + 40 = 60 \\
 5 + 7 = 12 \\
 60 + 12 = 72
 \end{array}$$

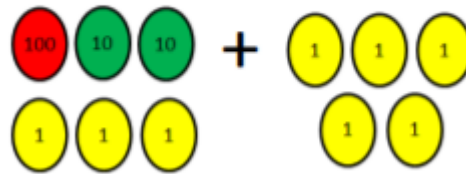


Addition of HTO + O without regrouping

$$123 + 5 = 128$$



$$123 + 5 = 128$$



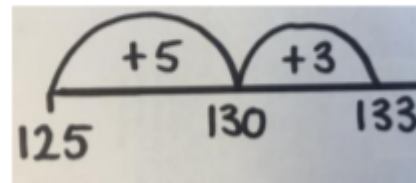
$$123 + 5 = 120 + 8$$

Addition of HTO + O with regrouping

$$125 + 8 = 133$$



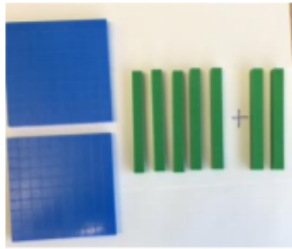


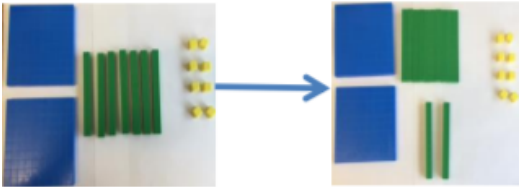
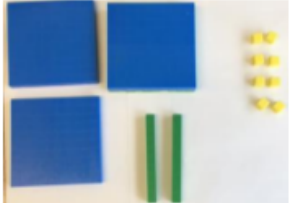
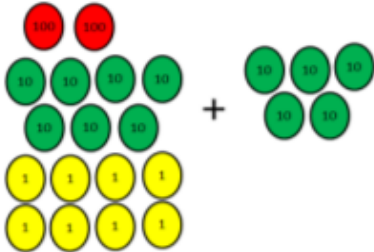
$$125 + 8 = 133$$

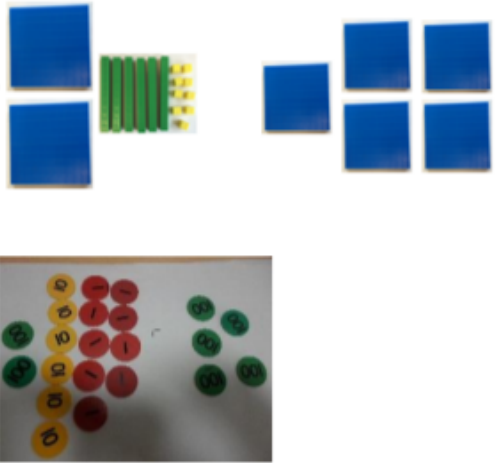
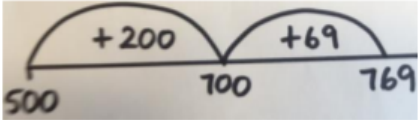
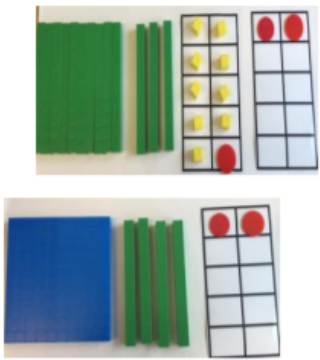
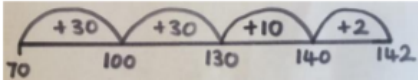
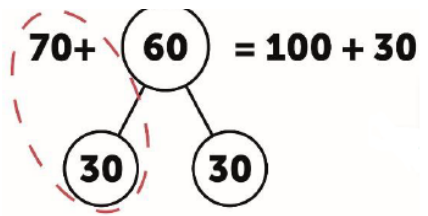


$$125 + 8 = 133$$

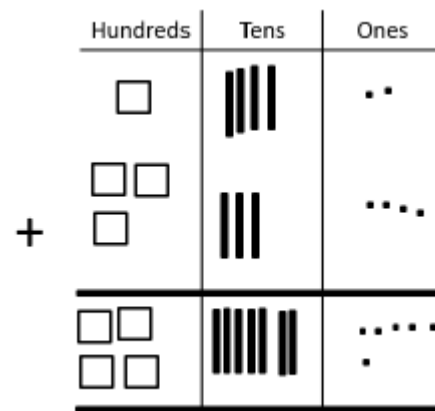
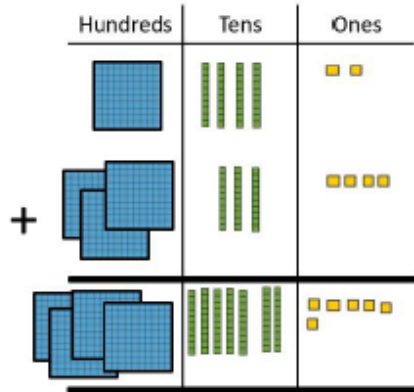
$$125 + 5 + 3 = 133$$



<p>Addition of HTO + T without regrouping</p>	<p><b>250 + 20 = 270</b></p>  	<p><b>250 + 20 = 270</b></p> 	<p><b>250 + 20 = 200 + 70</b></p> <p><i>Leading to any HTO + multiple of 10 (not crossing the ten boundary)</i></p> <p><b>234 + 30 = 200 + 60 + 4</b></p>
<p>Addition of HTO + T with regrouping</p>	<p><b>278 + 50 = 328</b></p>  <p><u>Children to understand the exchange of 10 tens for one hundred.</u></p> 	<p><b>278 + 50 = 328</b></p> 	<p><b>278 + 50 = 328</b></p> <p><b>270 + 50 + 8 = 328</b></p>

<p>Addition of HTO + H</p>	<p><b>269 + 500 = 769</b></p>  <p>The top image shows base ten blocks representing 269 (two hundreds, six tens, nine ones) and 500 (five hundreds). The result is 769 (seven hundreds, six tens, nine ones). The bottom image shows a place value chart with 2 hundreds, 6 tens, and 9 ones in the first column, and 5 hundreds in the second column. The result is 7 hundreds, 6 tens, and 9 ones.</p>	<p><b>269 + 500 = 769</b></p>  <p>A number line starting at 500, jumping to 700 (+200), and then to 769 (+69).</p>	<p><b>269 + 500 = 769</b>  <math>200 + 500 + 69 = 769</math></p>
<p>Addition of any TO + TO</p> <p>Using Partitioning</p>	<p><b>79 + 63 = 142</b></p>  <p>The top image shows base ten blocks for 79 (seven tens, nine ones) and 63 (six tens, three ones). The bottom image shows a place value chart with 7 tens and 9 ones in the first column, and 6 tens and 3 ones in the second column. The result is 14 tens and 2 ones.</p> <p>And the same with place value counters ensuring children understand the exchange.</p>	<p><b>79 + 63 = 142</b></p>  <p>A number line starting at 70, jumping to 100 (+30), then to 130 (+30), then to 140 (+10), and finally to 142 (+2).</p>	<p><b>79 + 63 = 142</b>  <math>70 + 60 = 130</math>  <math>9 + 3 = 9 + 1 + 2</math>  <math>130 + 12 = 142</math></p>  <p>A diagram showing 70 + 60 = 100 + 30. The 70 is circled in red, and the 60 is also circled in red. Below it, 30 and 30 are shown in circles, connected by lines to the 60 above. Below that, 100 + 30 = 130 is written.</p> <p><b>100 + 30 = 130</b></p>

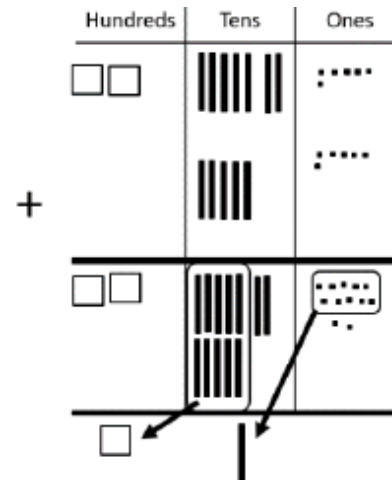
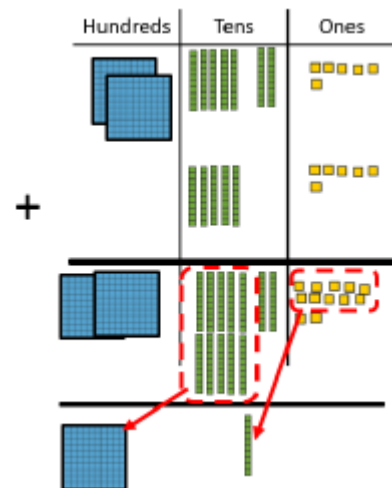
Add 3 digit numbers together without regrouping



$$\begin{array}{r} 142 \\ + 334 \\ \hline 476 \end{array}$$

$$142 + 334 = 476$$

Add 3 digit numbers together with regrouping



$$\begin{array}{r} 276 \\ + 56 \\ \hline 332 \end{array}$$

$$276 + 56 = 332$$

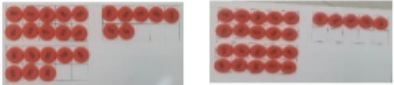

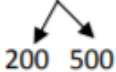
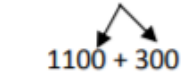
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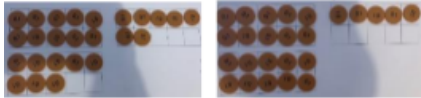


**Year 4 - Addition**

Pupils should be taught to:

- Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate
- Estimate and use inverse operations to check answers to a calculation
- Solve addition two-step problems in contexts, deciding which operations and methods to use and why

*Bar models to be used to support decision making and where the missing numbers fit in our calculations.*

<p>Adding a multiple of 1000 or 100 to a 4 digit number</p>		<p>1700 + 1400</p> 	<p>1800 + 700</p>  <p>1700 + 1400</p> 
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<p>Mental calculations (rounding, doubling, using number bonds, adding near doubles)</p>			$  \begin{array}{c}  1376 + 1598 \\  \swarrow \quad \searrow \\  1374 + 2 \quad \longrightarrow \quad 1374 + 1600  \end{array}  $
<p>Add numbers to one decimal place</p>	<p>Add numbers to one decimal place <math>1.8 + 0.7</math></p> 	<p><math>1.8 + 0.7</math></p> 	<p><math>1.8 + 0.7</math></p> 

Add numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.

Use place value equipment on a place value grid to organise thinking.

Ensure that children understand how the columns relate to place value and what to do if the numbers are not all 4-digit numbers.

Use equipment to show  $1,905 + 775$ .

Th	H	T	O
1	9	0	5
	7	7	5

Why have only three columns been used for the second row? Why is the Thousands box empty?

Which columns will total 10 or more?

Use place value equipment to model required exchanges.

Th	H	T	O
1	9	0	5
	7	7	5

Th	H	T	O
1	9	0	5
	7	7	5

Th	H	T	O
1	9	0	5
	7	7	5

Th	H	T	O
2	7	9	1

Include examples that exchange in more than one column.

Use a column method to add, including exchanges.

Th	H	T	O
1	5	5	4
+ 4	2	3	7
<hr/>			
		9	1

Th	H	T	O
1	5	5	4
+ 4	2	3	7
<hr/>			
		9	1

Th	H	T	O
1	5	5	4
+ 4	2	3	7
<hr/>			
7	9	1	

Th	H	T	O
1	5	5	4
+ 4	2	3	7
<hr/>			
5	7	9	1

Include examples that exchange in more than one column.

## Addition Year 5

Pupils should be taught to:

- Add whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- Add numbers mentally with increasingly large numbers
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

*Bar models to be used to support decision making and where the missing numbers fit in our calculations.*

Mental calculations (rounding, doubling, using number bonds, adding near doubles)

Examples

**1445 + 2999**  
 $1445 + 3000 - 1$

**1299 + 1299**  
Double 1300 - 2

**443 + 445**  
Near double  $443 + 2$

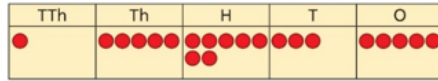
**12.36 + 5.24**  
 $0.36 + 0.24 = 0.6$   
 $17 + 0.6 = 17.6$

**36.25 + 23.43**  
Add each place value column individually

Addition of two numbers (more than 4 digits) using column addition.

Use place value equipment to represent additions.

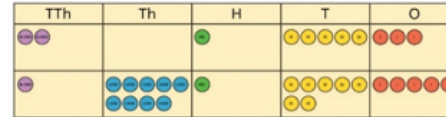
Add a row of counters onto the place value grid to show  $15,735 + 4,012$ .



MacDonalds sold £9957.68 worth of hamburgers and £1238.5 worth of chicken nuggets. How much money did they take altogether?



Represent additions, using place value equipment on a place value grid alongside written methods.



I need to exchange 10 tens for a 100.

TTh	Th	H	T	O
2	0	1	5	3
+	1	9	1	7
3	9	3	2	8

Formal method (using carrying) with more than four digits

$$\begin{array}{r} 55825 \\ + 37486 \\ \hline 93312 \\ \hline \end{array}$$

$$\begin{array}{r} 75879 \\ + 9486 \\ \hline 85365 \\ \hline \end{array}$$

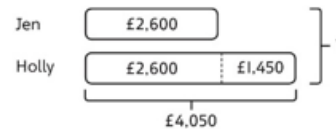
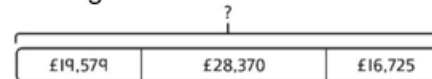
Use formal method to solve two-step problems in contexts.

$$\begin{array}{r} 12.73 \\ + 8.39 \\ \hline 21.12 \\ \hline \end{array}$$

$$\begin{array}{r} £44.73 \\ + £8.39 \\ \hline £53.12 \\ \hline \end{array}$$

Representing Additions

Bar models represent addition of two or more numbers in the context of problem solving.



Th	H	T	O
2	6	0	0
+	1	4	5
4	0	5	0

Th	H	T	O
2	6	0	0
+	4	0	5
6	6	5	0


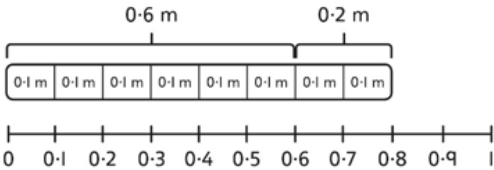


Use approximation to check whether answers are reasonable.

TTh	Th	H	T	O
2	3	4	0	5
+	7	8	9	2
2	0	2	9	7

TTh	Th	H	T	O
2	3	4	0	5
+	7	8	9	2
3	1	2	9	7

I will use  $23,000 + 8,000$  to check.



<p>Adding Tenths</p>	<p>Link measure with addition of decimals.</p> <p><i>Two lengths of fencing are 0.6 m and 0.2 m. How long are they when added together?</i></p> 	<p>Use a bar model with a number line to add tenths.</p>  <p><math>0.6 + 0.2 = 0.8</math> <i>6 tenths + 2 tenths = 8 tenths</i></p>	<p>Understand the link with adding fractions.</p> $\frac{6}{10} + \frac{2}{10} = \frac{8}{10}$ <p><i>6 tenths + 2 tenths = 8 tenths</i> <math>0.6 + 0.2 = 0.8</math></p>
<p>Adding decimals using column addition</p>	<p>Use place value equipment to represent additions.</p> <p><i>Show <math>0.23 + 0.45</math> using place value counters.</i></p>	<p>Use place value equipment on a place value grid to represent additions.</p> <p>Represent exchange where necessary.</p>  <p>Include examples where the numbers of decimal places are different.</p> 	<p>Add using a column method, ensuring that children understand the link with place value.</p> $\begin{array}{r} \text{O} \cdot \text{Tth} \text{Hth} \\ 0 \cdot 23 \\ + 0 \cdot 45 \\ \hline 0 \cdot 68 \end{array}$ <p>Include exchange where required, alongside an understanding of place value.</p> $\begin{array}{r} \text{O} \cdot \text{Tth} \text{Hth} \\ 0 \cdot 92 \\ + 0 \cdot 33 \\ \hline 1 \cdot 25 \end{array}$ <p>Include additions where the numbers of decimal places are different.</p> <p><math>3.4 + 0.65 = ?</math></p> $\begin{array}{r} \text{O} \cdot \text{Tth} \text{Hth} \\ 3 \cdot 40 \\ + 0 \cdot 65 \\ \hline \end{array}$

## Addition Year 6

Pupils should be taught to:

- perform mental calculations, using increasingly large numbers
- use their knowledge of the order of operations to carry out calculations involving the 4 operations
- solve addition multi-step problems in contexts, deciding which methods to use and why solve problems involving addition

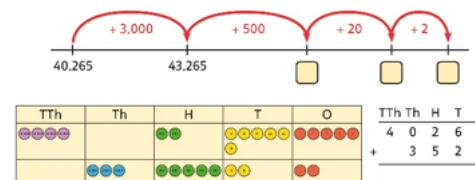
*Bar models to be used to support decision making and where the missing numbers fit in our calculations*

### Comparing and selecting efficient methods

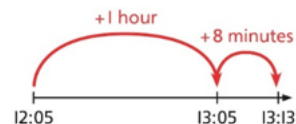
Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.



Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation. Compare written and mental methods alongside place value representations.



Use bar model and number line representations to model addition in problem-solving and measure contexts.



Use column addition where mental methods are not efficient. Recognise common errors with column addition.

$$32,145 + 4,302 = ?$$

	TTh	Th	H	T	O
	3	2	1	4	5
+		4	3	0	2
	3	6	4	4	7


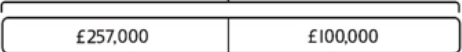

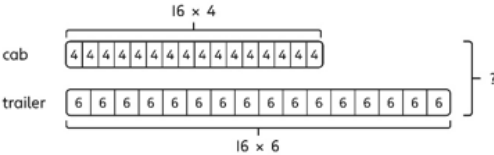
	TTh	Th	H	T	O
	3	2	1	4	5
+	4	3	0	2	
	7	5	1	6	5

*Which method has been completed accurately?*

*What mistake has been made?*

Column methods are also used for decimal additions where mental methods are not efficient.

	H	T	O	·	Tth	Hth
	1	4	0	·	0	9
+		4	9	·	8	9
	1	8	9	·	9	8

<p><b>Selecting mental methods for larger numbers where appropriate</b></p>	<p>Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.</p>  <p><math>2,411,301 + 500,000 = ?</math></p> <p><i>This would be 5 more counters in the HTh place.</i></p> <p><i>So, the total is 2,911,301.</i></p> <p><math>2,411,301 + 500,000 = 2,911,301</math></p>	<p>Use a bar model to support thinking in addition problems.</p> <p><math>257,000 + 99,000 = ?</math></p>  <p><i>I added 100 thousands then subtracted 1 thousand.</i></p> <p><math>257 \text{ thousands} + 100 \text{ thousands} = 357 \text{ thousands}</math></p> <p><math>257,000 + 100,000 = 357,000</math>  <math>357,000 - 1,000 = 356,000</math></p> <p><i>So, <math>257,000 + 99,000 = 356,000</math></i></p>	<p>Use place value and unitising to support mental calculations with larger numbers.</p> <p><math>195,000 + 6,000 = ?</math></p> <p><math>195 + 5 + 1 = 201</math></p> <p><i>195 thousands + 6 thousands = 201 thousands</i></p> <p><i>So, <math>195,000 + 6,000 = 201,000</math></i></p>
<p><b>Understanding order of operations in calculations</b></p>	<p>Use equipment to model different interpretations of a calculation with more than one operation. Explore different results.</p> <p><math>3 \times 5 - 2 = ?</math></p>  <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p><math>3 \times 5 - 2</math></p> <p>↓ ↓</p> <p><math>3 \times 3 = 9</math></p> </div> <div style="text-align: center;"> <p><math>3 \times 5 - 2</math></p> <p>↓ ↓</p> <p><math>15 - 2 = 13</math></p> </div> </div>	<p>Model calculations using a bar model to demonstrate the correct order of operations in multi-step calculations.</p>  <p>This can be written as: <math>16 \times 4 + 16 \times 6</math></p> <p><math>64 + 96 = 160</math></p>	<p>Understand the correct order of operations in calculations without brackets.</p> <p>Understand how brackets affect the order of operations in a calculation.</p> <p><math>4 + 6 \times 16</math>  <math>4 + 96 = 100</math></p> <p><math>(4 + 6) \times 16</math>  <math>10 \times 16 = 160</math></p>

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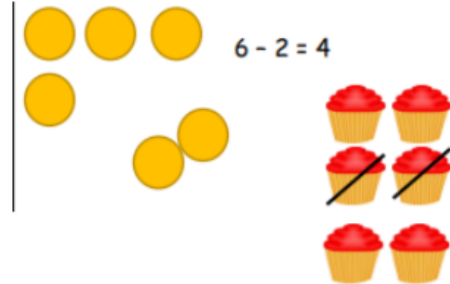
## Year 1 Subtraction

Pupils should be taught to:

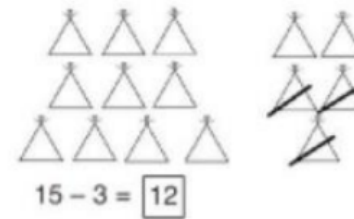
- read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as  $7 = \square - 9$ .

Taking away ones

Use physical objects, counters, cubes etc. to show how objects can be taken away.



Cross out drawn objects to show what has been taken away.



Show subtraction number sentence.

$$18 - 3 = 15$$

$$8 - 2 = 6$$

## Counting back

Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.

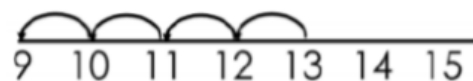
$$13 - 4 =$$



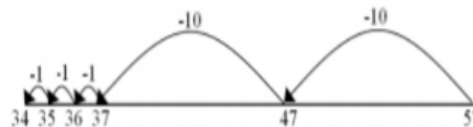
Use counters and move them away from the group as you take them away counting backwards as you go.



Count back on a number line or number track.



Start at the bigger number and count back the smaller number showing the jumps on the number line.



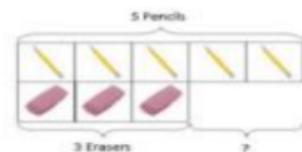
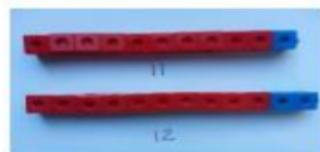
This can progress all the way to counting back using two 2 digit numbers.

Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

$$13 - 4 =$$

## Find the difference

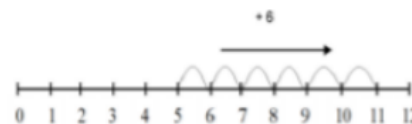
Compare amounts and objects to find the difference.



Use cubes to build towers or make bars to find the difference.

Use basic bar models with items to find the difference

Count on to find the difference.



Draw bars to find the difference between 2 numbers.

### Comparison Bar Models

Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.

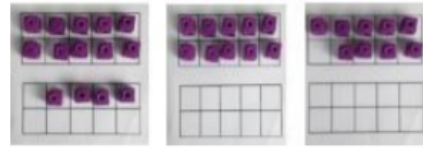


Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.

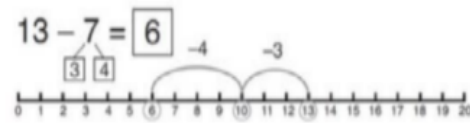
$$23 - 15 =$$

## Part Part Whole Model

Make 14 on the ten frame.  
Take away the four first to make 10 and then  
takeaway one more so you have taken away 5.  
You are left with the answer of 9.



14 - 5 =



Start at 13. Take away 3 to reach 10. Then take  
away the remaining 4 so you have taken away 7  
altogether. You have reached your answer.

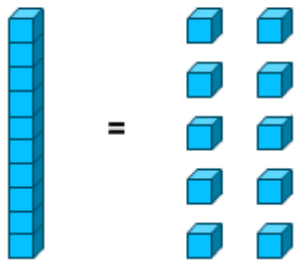
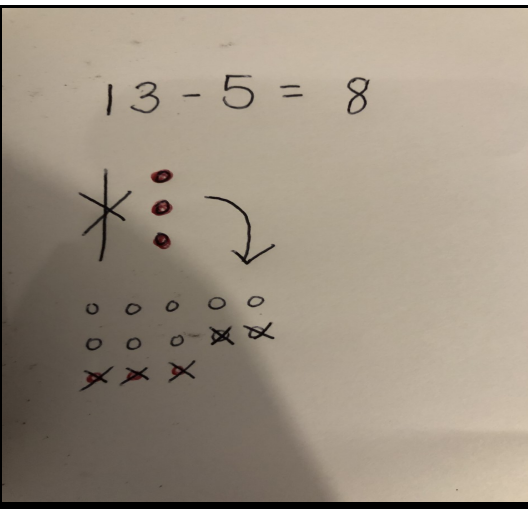
How many do we take off to reach the next 10?  
How many do we have left to take off?

16 - 8 =

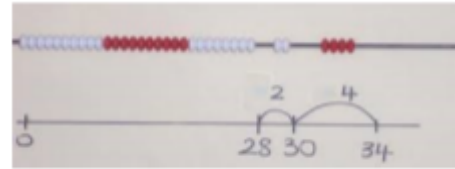
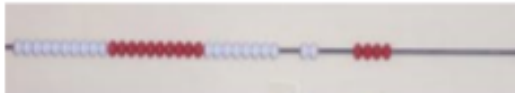
## Year 2 Subtraction

Pupils should be able to:

- solve problems with subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100
- subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers

Objectives and strategies	Concrete	Pictorial	Abstract
Regroup a ten into ten ones and subtract.	<p>1 ten      10 ones</p>  <p>A vertical rod of 10 blue cubes is shown on the left, labeled '1 ten'. To its right is an equals sign, followed by two columns of 5 blue cubes each, labeled '10 ones'.</p>	 <p>A photograph of a whiteboard showing the subtraction 13 - 5 = 8. The equation is written at the top. Below it, a ten-frame is drawn with 10 dots. The top row has 3 dots, and the bottom row has 5 dots. A large asterisk is drawn over the top row, and an arrow points from the top row to the bottom row, indicating the regrouping process. The bottom row has 2 dots crossed out with an 'X'.</p>	$13 - 5 = 8$

Make ten strategy  
Progression  
should be crossing  
one ten, crossing  
more than one  
ten, crossing the  
hundreds.



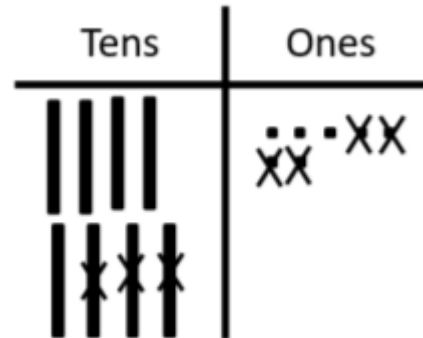
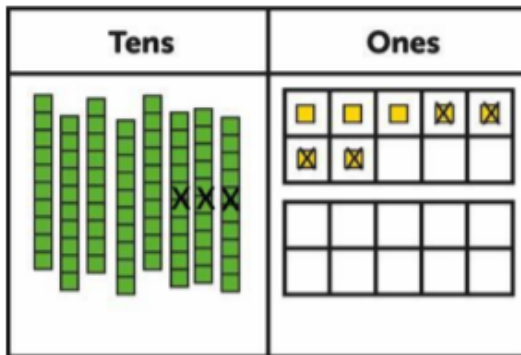
$$34 - 28$$

Use a bead bar or bead strings to model counting to next ten and the rest.

Subtract two two  
digit numbers  
without  
regrouping.

Make the two-digit number using tens and ones and then take away the amount to be subtracted.

$$87 - 34 = 53$$



When confident, children will begin to use the expanded written method to subtract tens and ones.

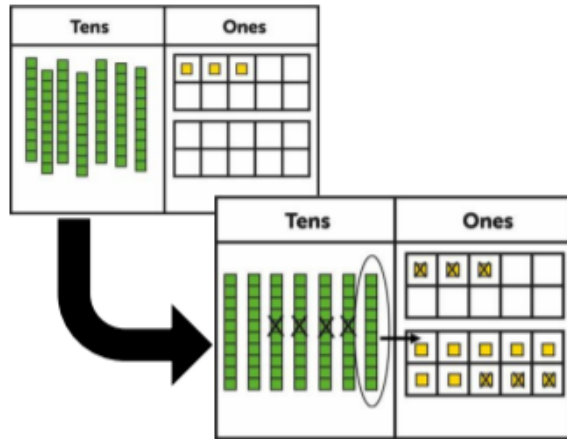
$$\begin{array}{r} 80 + 7 \\ - 30 + 4 \\ \hline 50 + 3 \end{array}$$

$$87 - 34 = 53$$

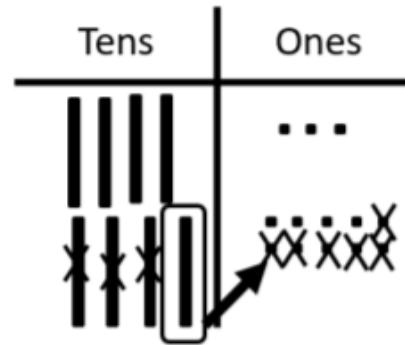


Subtract two digit numbers with regrouping.

Make tens and ones and then regroup one ten for ten ones to be able to subtract the number.



Children will assess if they need to use regrouping to solve the two digit subtraction question. Draw the method they need to use.



When confident, children will begin to use the expanded written method to subtract tens and ones.

$$\begin{array}{r} 60 \quad 13 \\ 70 + 3 \\ - 40 + 6 \\ \hline 20 + 7 \end{array}$$

$$73 - 46 = 27$$


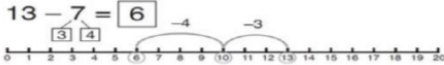
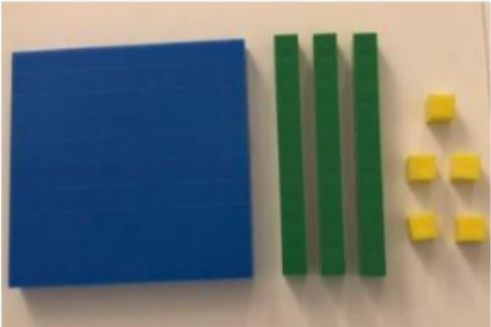
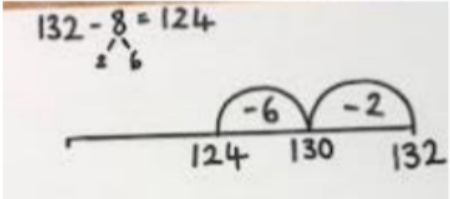
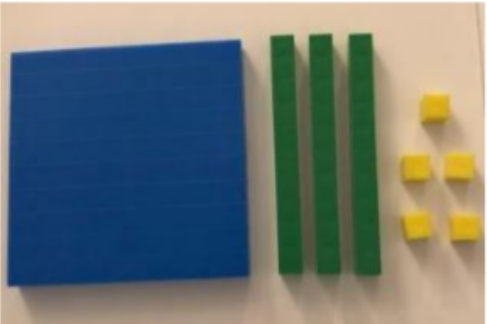
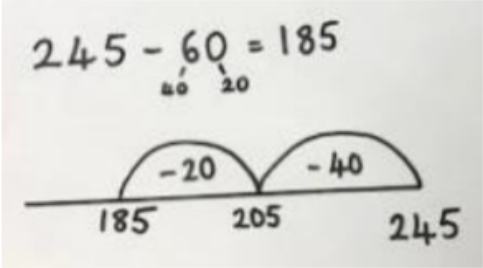
### Subtraction Year 3

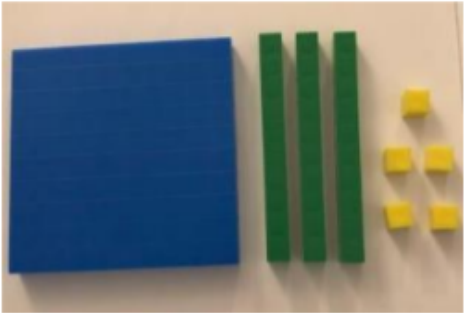
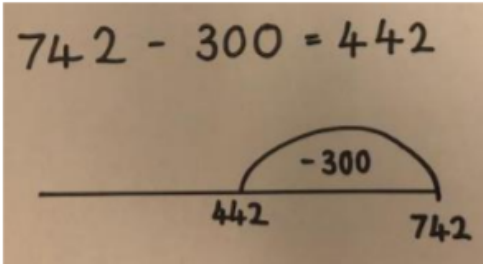
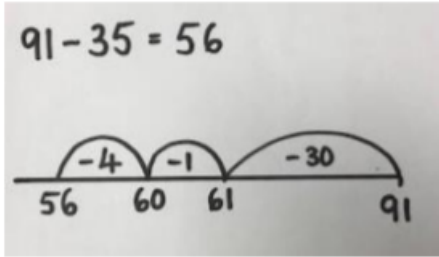
Pupils should be taught to:

- subtract numbers mentally, including:
  - a three-digit number and 1s
  - a three-digit number and 10s
  - a three-digit number and 100s
- subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction
- solve problems, including missing number problems, using number facts, place value, and more complex subtraction

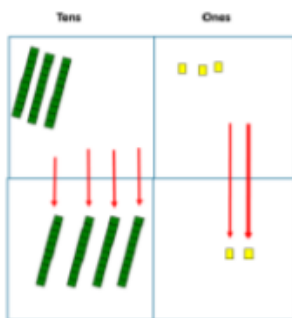
Children should also be taught to calculate the difference when two numbers are close in range e.g.  $114 - 98$ , counting on  $98 + 2 = 100$  then  $100 + 14 = 114$ , therefore the difference is 16 at all stages.

*Bar models to be used to support decision making and where the missing numbers fit in our calculations.*

<p>Make 10</p>	<p><math>14 - 9 =</math></p> 	<p><math>13 - 7 = 6</math></p>  <p>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</p>	<p><math>16 - 8 =</math></p> <p>How many do we take off to reach the next 10?</p> <p>How many do we have left to take off?</p>
<p>Subtract HTO – O (using bonds leading to partitioning)</p>	<p><math>135 - 2 = 133</math></p> 		<p><math>148 - 5 = 143</math></p> <p><math>152 - 2 = 150</math></p> <p><math>150 - 5 = 145</math></p> <p><math>152 - 7 = 152 - 2 - 5 = 145</math></p>
<p>Subtract HTO – T (using bonds leading to partitioning)</p>	<p><math>135 - 20 = 115</math></p> 		<p><math>248 - 20 = 228</math></p> <p><math>162 - 70 = 92</math></p> <p><math>162 - 60 = 102</math></p> <p><math>102 - 10 = 92</math></p>

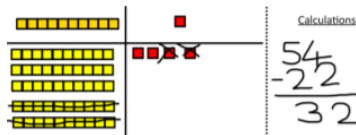
<p>Subtract HTO – H (using bonds)</p>	<p><math>635 - 400 = 235</math></p> 	 <p><math>742 - 300 = 442</math></p>	<p><math>478 - 200 = 278</math></p>
<p>Subtract any TO – TO Using partitioning</p>	<p><math>72 - 26 = 46</math></p> <p>Use dienes and place value counters to model <math>72 - 20 - 2 - 4 = 46</math></p>	 <p><math>91 - 35 = 56</math></p>	<p><math>78 - 49 = 29</math></p> <p><math>78 - 40 = 38</math>  <math>38 - 8 = 30</math>  <math>30 - 1 = 29</math></p> <p><math>78 - 40 - 8 - 1 = 29</math></p>

Column method without regrouping



Use Base 10 to make the bigger number then take the smaller number away.

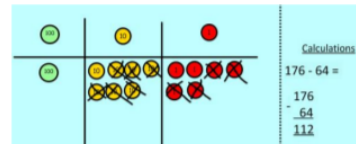
Show how you partition numbers to subtract. Again make the larger number first.



Calculations

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

Draw the Base 10 or place value counters alongside the written calculation to help to show working.



Calculations

$$\begin{array}{r} 176 \\ - 64 \\ \hline 112 \end{array}$$

$$47 - 24 = 23$$

$$\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$$

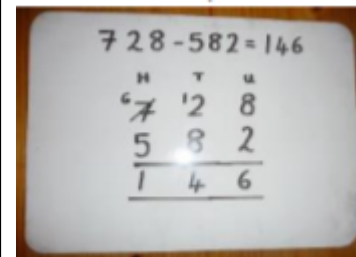
This will lead to a clear

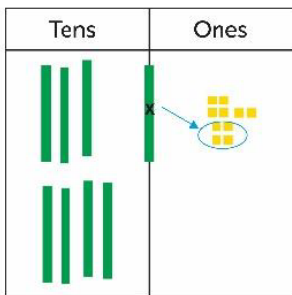
written column subtraction.

Column method with regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters






Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.


Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Now I can subtract my ones.


Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.

$$\begin{array}{r} \overset{7}{\cancel{8}} \overset{1}{0} \\ - 24 \\ \hline 6 \\ \hline \end{array}$$

## Subtraction Year 4

Pupils should be taught to:

- subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve subtraction two-step problems in contexts, deciding which operations and methods to use and why

*Bar models to be used to support decision making and where the missing numbers fit in our calculations.*

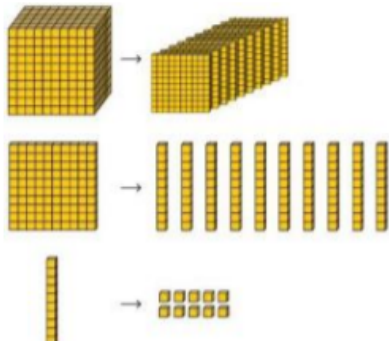
Subtraction of  
multiples  
10/100/1000



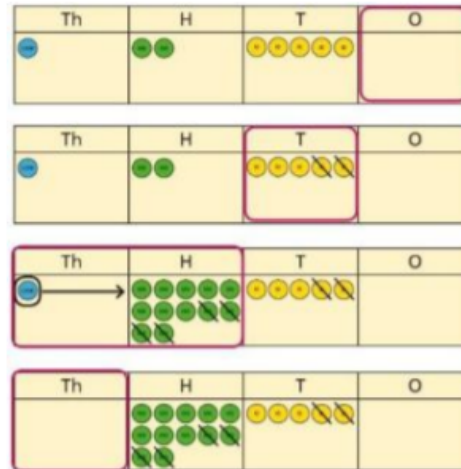
$$2600 - 800$$

Subtract with numbers up to four digits, including exchanging

Understand why exchange of a 1,000 for 100s, a 100 for 10s, or a 10 for 1s may be necessary.



Represent place value equipment on a place value grid to subtract, including exchanges where needed.



Use column subtraction, with understanding of the place value of any exchange required.

$$\begin{array}{r} \text{Th} \quad \text{H} \quad \text{T} \quad \text{O} \\ 1 \quad 2 \quad 5 \quad 0 \\ - \quad 4 \quad 2 \quad 0 \\ \hline \quad \quad \quad 0 \end{array}$$

$$\begin{array}{r} \text{Th} \quad \text{H} \quad \text{T} \quad \text{O} \\ 1 \quad 2 \quad 5 \quad 0 \\ - \quad 4 \quad 2 \quad 0 \\ \hline \quad \quad 3 \quad 0 \end{array}$$

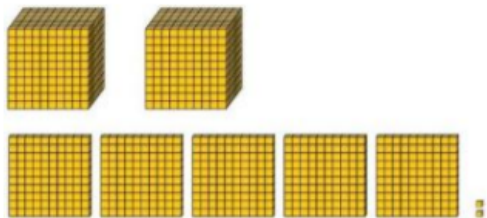
$$\begin{array}{r} \text{Th} \quad \text{H} \quad \text{T} \quad \text{O} \\ \cancel{1} \quad 2 \quad 5 \quad 0 \\ - \quad 4 \quad 2 \quad 0 \\ \hline 8 \quad 3 \quad 0 \end{array}$$

$$\begin{array}{r} \text{Th} \quad \text{H} \quad \text{T} \quad \text{O} \\ \cancel{1} \quad 2 \quad 5 \quad 0 \\ - \quad 4 \quad 2 \quad 0 \\ \hline 8 \quad 3 \quad 0 \end{array}$$

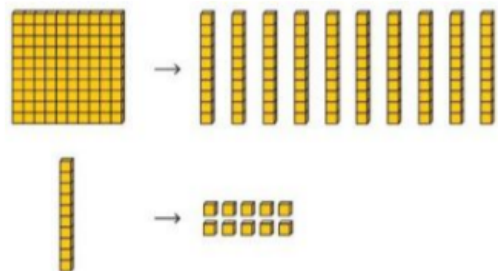
Column subtraction with exchange across more than one column

Understand why two exchanges may be necessary.

$$2,502 - 243 = ?$$

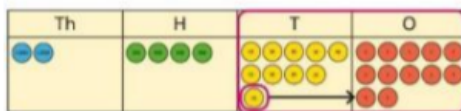
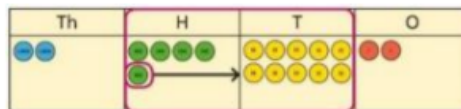


*I need to exchange a 10 for some 1s, but there are not any 10s here.*



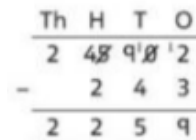
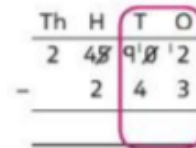
Make exchanges across more than one column where there is a zero as a place holder.

$$2,502 - 243 = ?$$



Make exchanges across more than one column where there is a zero as a place holder.

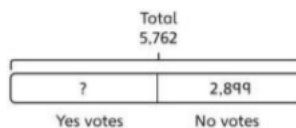
$$2,502 - 243 = ?$$





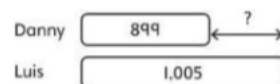
Representing subtractions and checking strategies

Use bar models to represent subtractions where a part needs to be calculated.



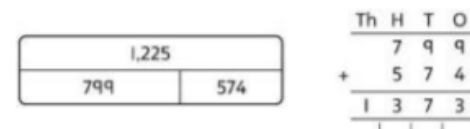
*I can work out the total number of Yes votes using  $5,762 - 2,899$ .*

Bar models can also represent 'find the difference' as a subtraction problem.



Use inverse operations to check subtractions.

*I calculated  $1,225 - 799 = 574$ .  
I will check by adding the parts.*

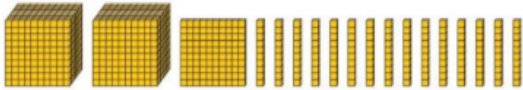
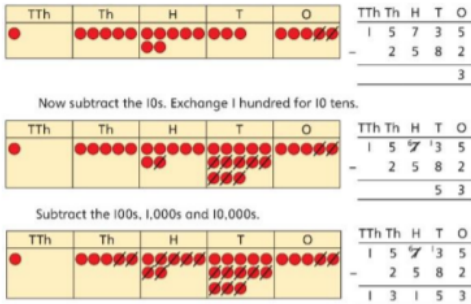


*The parts do not add to make 1,225.  
I must have made a mistake.*

## Subtraction Year 5

Pupils should be taught to:

- subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)
- subtract numbers mentally with increasingly large numbers
- solve subtraction multi-step problems in contexts, deciding which methods to use and why

<p>Column subtraction with whole numbers</p>	<p>Use place value equipment to understand where exchanges are required.</p> <p><math>2,250 - 1,070</math></p> 	<p>Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required.</p> <p><math>15,735 - 2,582 = 13,153</math></p> 	<p>Use column subtraction methods with exchange where required.</p> $\begin{array}{r} \text{TTh Th H T O} \\ \cancel{5} \cancel{1} \cancel{2} \cancel{1} 0 \quad 9 \quad 7 \\ - 1 \quad 8 \quad 5 \quad 3 \quad 4 \\ \hline 4 \quad 3 \quad 5 \quad 6 \quad 3 \end{array}$ <p><math>62,097 - 18,534 = 43,563</math></p>
<p>Checking strategies and representing subtractions</p>		<p>Bar models represent subtractions in problem contexts, including 'find the difference'.</p> <p>Athletics Stadium <span style="border: 1px solid black; padding: 2px 20px;">75,450</span></p> <p>Hockey Centre <span style="border: 1px solid black; padding: 2px 20px;"> </span> ← <math>42,300</math> →</p> <p>Velodrome <span style="border: 1px solid black; padding: 2px 20px;">15,735</span> ← <math>?</math> →</p>	<p>Children can explain the mistake made when the columns have not been ordered correctly.</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid gray; padding: 5px;"> <p><b>Bella's working</b></p> <math display="block">\begin{array}{r} \text{TTh Th H T O} \\ 1 \quad 7 \quad 8 \quad 7 \quad 7 \\ + 4 \quad 0 \quad 1 \quad 2 \\ \hline 5 \quad 7 \quad 9 \quad 9 \quad 7 \end{array}</math> </div> <div style="border: 1px solid gray; padding: 5px;"> <p><b>Correct method</b></p> <math display="block">\begin{array}{r} \text{TTh Th H T O} \\ 1 \quad 7 \quad 8 \quad 7 \quad 7 \\ + 4 \quad 0 \quad 1 \quad 2 \\ \hline 2 \quad 1 \quad 8 \quad 8 \quad 9 \end{array}</math> </div> </div> <p>Use approximation to check calculations.</p> <p><i>I calculated <math>18,000 + 4,000</math> mentally to check my subtraction.</i></p>

## Subtraction Year 6

Pupils should be taught to:

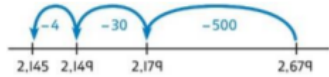
- perform mental calculations, including with increasingly large numbers
- use their knowledge of the order of operations to carry out calculation involving the 4 operations
- solve subtraction multi-step problems in contexts, deciding which methods to use and why
- solve problems using subtraction

Comparing and selecting efficient methods

Use counters on a place value grid to represent subtractions of larger numbers.

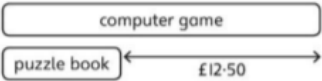


Compare subtraction methods alongside place value representations.



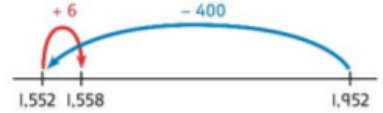
$$\begin{array}{r} \text{Th H T O} \\ 2 \ 6 \ 7 \ 9 \\ - \ 5 \ 3 \ 4 \\ \hline 2 \ 1 \ 4 \ 5 \end{array}$$

Use a bar model to represent calculations, including 'find the difference' with two bars as comparison.



Compare and select methods. Use column subtraction when mental methods are not efficient. Use two different methods for one calculation as a checking strategy.

$$\begin{array}{r} \text{Th H T O} \\ 1 \ 5 \ 5 \ 8 \\ - \ 1 \ 5 \ 5 \ 8 \\ \hline 3 \ 9 \ 4 \end{array}$$



Use column subtraction for decimal problems, including in the context of measure.

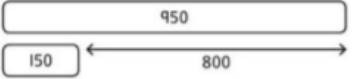
$$\begin{array}{r} \text{H T O} \cdot \text{Tth Hth} \\ 3 \ 0 \ 9 \cdot 6 \ 0 \\ - 2 \ 0 \ 6 \cdot 4 \ 0 \\ \hline 1 \ 0 \ 3 \cdot 2 \ 0 \end{array}$$

Subtracting mentally with larger numbers

Use a bar model to show how unitising can support mental calculations.

Use a bar model to show how unitising can support mental calculations.

$950,000 - 150,000$   
That is 950 thousands - 150 thousands



So, the difference is 800 thousands.  
 $950,000 - 150,000 = 800,000$

Subtract efficiently from powers of 10.  
 $10,000 - 500 = ?$

## Year 1 Multiplication

Pupils should be taught to:

- solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

### Doubling

Use practical activities to show how to double a number.



double 4 is 8  
 $4 \times 2 = 8$

Draw pictures to show how to double a number.

Double 4 is 8



### Show calculation

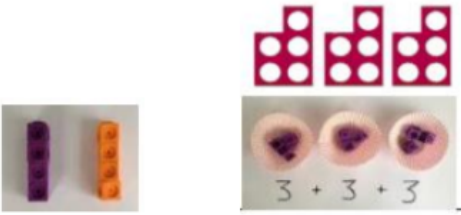

$$3 \times 2 = 6$$

### Counting in multiples



Count in multiples supported by concrete objects in equal groups.

Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30

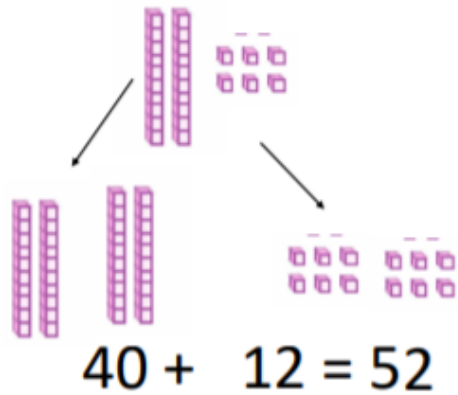
<p>Repeated addition</p>	<p>Use different objects to add equal groups.</p> 		<p>Write addition sentences to describe objects and pictures.</p> 
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## Year 2 Multiplication

Pupils should be able to:

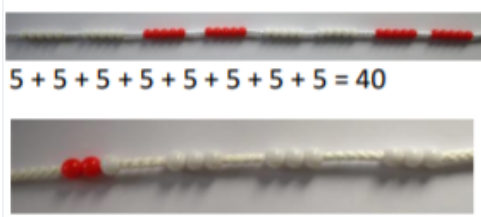
- recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers calculate mathematical statements for multiplication a within the multiplication tables and write them using the multiplication ( $\times$ ) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Objective and strategy	Concrete	Pictorial	Abstract
Doubling	<p>Model doubling using base ten by partitioning tens and ones. Then double the tens and double the ones. Make the total amount.</p> <p>Double 26 : 26 x 2 =</p>	Draw pictures and representations to show how to double numbers.	26 x 2

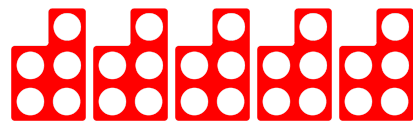


Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)

Using different manipulatives to make repeated additions sums.

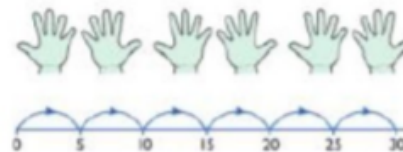


$3 + 3 + 3 + 3 = 12$   
 $3 \times 4 = 12$



$5 + 5 + 5 + 5 + 5 = 25$   
 $5 \times 5 = 25$

Number lines, counting sticks and bar models should be used to show representation of counting in multiples.



Count in multiples of a number aloud.

Write sequences with multiples of numbers.

0, 2, 4, 6, 8, 10

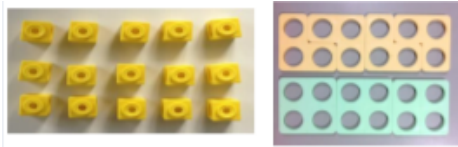
0, 3, 6, 9, 12, 15

0, 5, 10, 15, 20, 25, 30

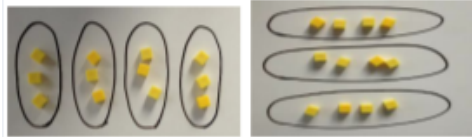
$4 \times 3 = \square$

Multiplication is commutative.

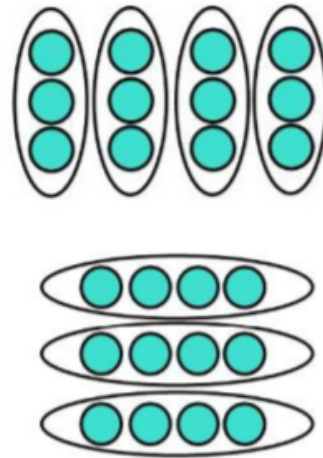
Create arrays using counters, numicon and cubes.



Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.



Use representations of arrays to show different calculations and explore commutativity.



$$12 = 3 \times 4$$

$$12 = 4 \times 3$$

Use an array to write multiplication sentences and reinforce repeated addition.



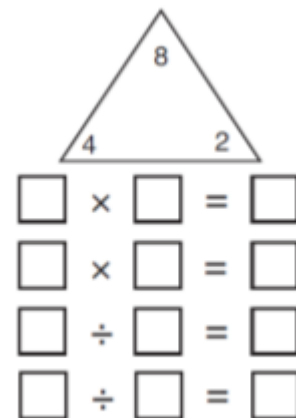
$$5 + 5 + 5 = 15$$

$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

Using the Inverse  
This should be taught alongside division, so pupils learn how they work alongside each other.



$$2 \times 4 = 8$$

$$4 \times 2 = 8$$

$$8 \div 2 = 4$$

$$8 \div 4 = 2$$

$$8 = 2 \times 4$$

$$8 = 4 \times 2$$

$$2 = 8 \div 4$$

$$4 = 8 \div 2$$

Show all 8 related fact family sentences.



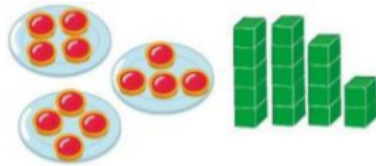
## Year 3 Multiplication

Pupils should be taught to:

- recall and use multiplication facts for the 3,4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

Understanding equal grouping and repeated addition

Children continue to build understanding of equal groups and the relationship with repeated addition. They recognise both examples and non-examples using objects.

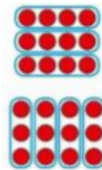


Children recognise that arrays can be used to model commutative multiplications.



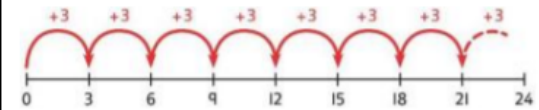
*I can see 3 groups of 8.  
I can see 8 groups of 3.*

Children recognise that arrays demonstrate commutativity.



*This is 3 groups of 4.  
This is 4 groups of 3.*

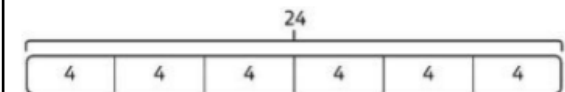
Children understand the link between repeated addition and multiplication.



*8 groups of 3 is 24.*

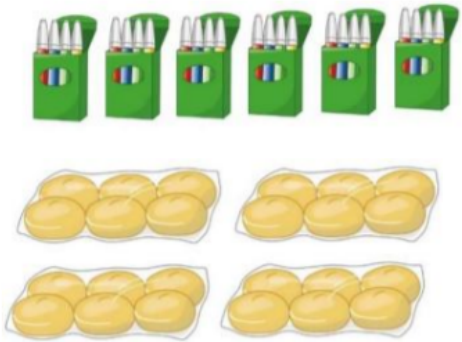
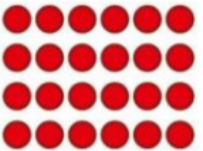

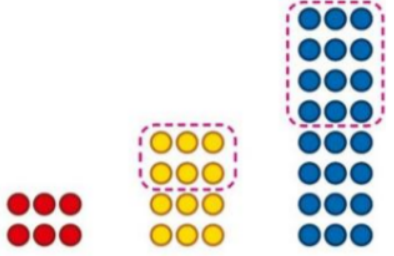
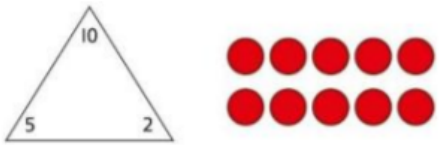
$$3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24$$
$$8 \times 3 = 24$$

A bar model may represent multiplications as equal groups.



$$6 \times 4 = 24$$



<p>Using commutativity to support understanding of the times tables</p>	<p>Understand how to use times-tables facts flexibly.</p>  <p><i>There are 6 groups of 4 pens. There are 4 groups of 6 bread rolls.</i></p> <p><i>I can use <math>6 \times 4 = 24</math> to work out both totals.</i></p>	<p>Understand how times-table facts relate to commutativity.</p>  <p><math>6 \times 4 = 24</math> <math>4 \times 6 = 24</math></p>	<p>Understand how times-table facts relate to commutativity.</p> <p><i>I need to work out 4 groups of 7.</i></p> <p><i>I know that <math>7 \times 4 = 28</math></i></p> <p><i>so, I know that</i></p> <p><i>4 groups of 7 = 28 and 7 groups of 4 = 28.</i></p>
<p>Understanding and using <math>\times 3</math>, <math>\times 2</math>, <math>\times 4</math> and <math>\times 8</math> tables.</p>	<p>Children learn the times-tables as 'groups of', but apply their knowledge of commutativity.</p>  <p><i>I can use the <math>\times 3</math> table to work out how many keys.</i> <i>I can also use the <math>\times 3</math> table to work out how many batteries.</i></p>	<p>Children understand how the <math>\times 2</math>, <math>\times 4</math> and <math>\times 8</math> tables are related through repeated doubling.</p>  <p><math>3 \times 2 = 6</math>      <math>3 \times 4 = 12</math>      <math>3 \times 8 = 24</math></p>	<p>Children understand the relationship between related multiplication and division facts in known times-tables.</p>  <p><math>2 \times 5 = 10</math> <math>5 \times 2 = 10</math> <math>10 \div 5 = 2</math> <math>10 \div 2 = 5</math></p>

Using known facts to multiply 10s, for example  $3 \times 40$

Explore the relationship between known times-tables and multiples of 10 using place value equipment.

Make 4 groups of 3 ones.



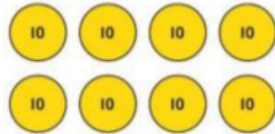
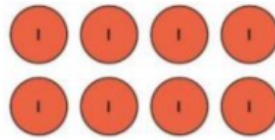
Make 4 groups of 3 tens.



What is the same?

What is different?

Understand how unitising 10s supports multiplying by multiples of 10.



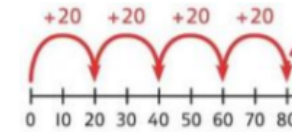
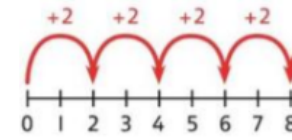
4 groups of 2 ones is 8 ones.

4 groups of 2 tens is 8 tens.

$$4 \times 2 = 8$$

$$4 \times 20 = 80$$

Understand how to use known times-tables to multiply multiples of 10.



$$4 \times 2 = 8$$

$$4 \times 20 = 80$$

Multiplying a 2-digit number by a 1-digit number

Understand how to link partitioning a 2-digit number with multiplying.

Each person has 23 flowers.

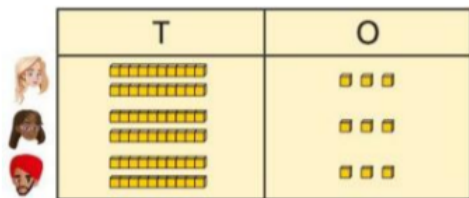
Each person has 2 tens and 3 ones.



There are 3 groups of 2 tens.

There are 3 groups of 3 ones.

Use place value equipment to model the multiplication context.

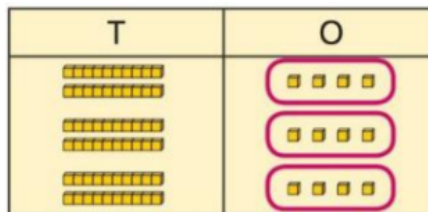


There are 3 groups of 3 ones.

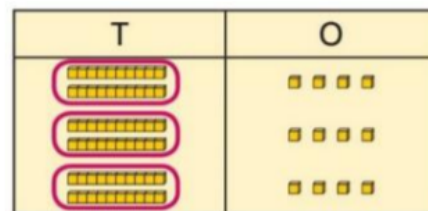
There are 3 groups of 2 tens.

Use place value to support how partitioning is linked with multiplying by a 2-digit number.

$$3 \times 24 = ?$$



$$3 \times 4 = 12$$



$$3 \times 20 = 60$$

$$60 + 12 = 72$$

$$3 \times 24 = 72$$

Use addition to complete multiplications of 2-digit numbers by a 1-digit number.

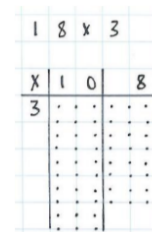
$$4 \times 13 = ?$$

$$4 \times 3 = 12$$

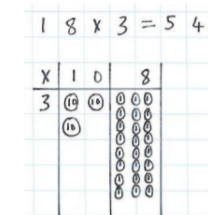
$$4 \times 10 = 40$$

$$12 + 40 = 52$$

$$4 \times 13 = 52$$



The two digit number is partitioned horizontally with the tens digit coming first. The equation is then represented using counters (or an array).



Again, the two digit number is partitioned horizontally with the tens digit coming first. This time the equation is represented using place value counters or Dienes.

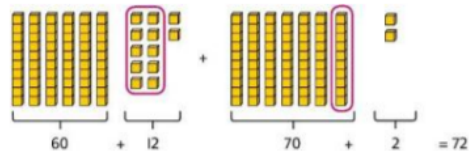
Multiplying a 2-digit number by a 1-digit number, expanded column method

Use place value equipment to model how 10 ones are exchanged for a 10 in some multiplications.

$$3 \times 24 = ?$$

$$3 \times 20 = 60$$

$$3 \times 4 = 12$$



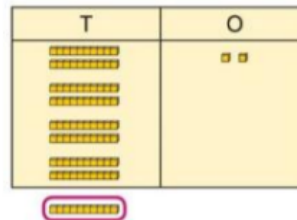
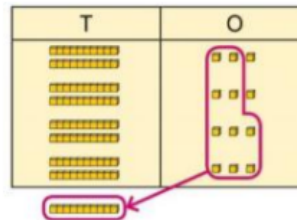
$$3 \times 24 = 60 + 12$$

$$3 \times 24 = 70 + 2$$

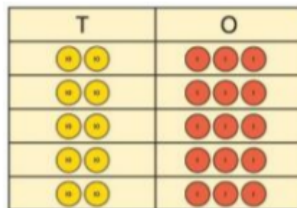
$$3 \times 24 = 72$$

Understand that multiplications may require an exchange of 1s for 10s, and also 10s for 100s.

$$4 \times 23 = ?$$



$$4 \times 23 = 92$$



$$5 \times 23 = ?$$

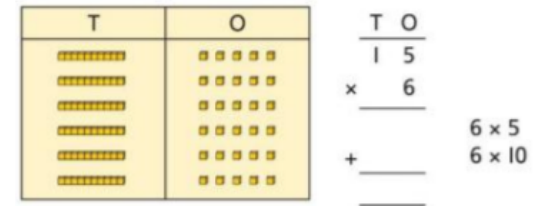
$$5 \times 3 = 15$$

$$5 \times 20 = 100$$

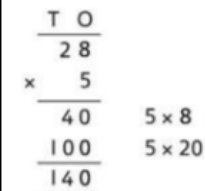
$$5 \times 23 = 115$$

Children may write calculations in expanded column form, but must understand the link with place value and exchange.

Children are encouraged to write the expanded parts of the calculation separately.



$$5 \times 28 = ?$$



## Year 4 Multiplication

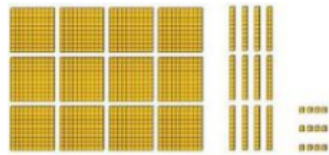
Pupils should be taught to:

- recall multiplication facts for multiplication tables up to 12 x 12
- use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together 3 numbers

- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

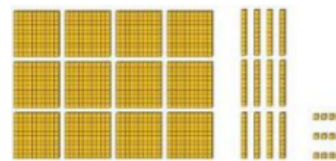
Multiplying by multiples of 10 and 100

Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.



*3 groups of 4 ones is 12 ones.  
 3 groups of 4 tens is 12 tens.  
 3 groups of 4 hundreds is 12 hundreds.*

Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.



$3 \times 4 = 12$   
 $3 \times 40 = 120$   
 $3 \times 400 = 1,200$

Use known facts and understanding of place value and commutativity to multiply mentally.


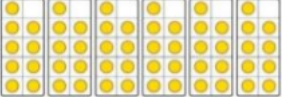


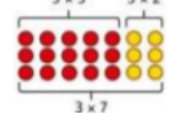

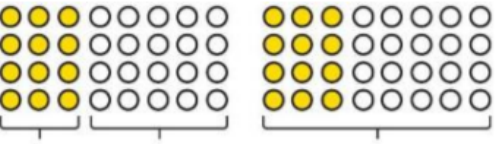
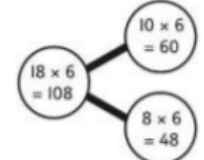
$$4 \times 7 = 28$$

$$4 \times 70 = 280$$

$$40 \times 7 = 280$$

$$4 \times 700 = 2,800$$

$$400 \times 7 = 2,800$$

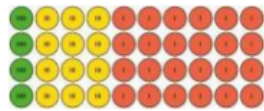
<p>Understanding times-tables up to <math>12 \times 12</math></p>	<p>Understand the special cases of multiplying by 1 and 0.</p>  <p><math>5 \times 1 = 5</math>                      <math>5 \times 0 = 0</math></p>	<p>Represent the relationship between the <math>\times 9</math> table and the <math>\times 10</math> table.</p>  <p>Represent the <math>\times 11</math> table and <math>\times 12</math> tables in relation to the <math>\times 10</math> table.</p>  <p><math>2 \times 11 = 20 + 2</math>  <math>3 \times 11 = 30 + 3</math>  <math>4 \times 11 = 40 + 4</math></p>  <p><math>4 \times 12 = 40 + 8</math></p>	<p>Understand how times-tables relate to counting patterns.</p> <p>Understand links between the <math>\times 3</math> table, <math>\times 6</math> table and <math>\times 9</math> table  <math>5 \times 6</math> is double <math>5 \times 3</math></p> <p><math>\times 5</math> table and <math>\times 6</math> table  <i>I know that <math>7 \times 5 = 35</math>  so I know that <math>7 \times 6 = 35 + 7</math>.</i></p> <p><math>\times 5</math> table and <math>\times 7</math> table  <math>3 \times 7 = 3 \times 5 + 3 \times 2</math></p>  <p><math>\times 9</math> table and <math>\times 10</math> table  <math>6 \times 10 = 60</math>  <math>6 \times 9 = 60 - 6</math></p>
<p>Understanding and using partitioning in multiplication</p>	<p>Make multiplications by partitioning.</p> <p><math>4 \times 12</math> is 4 groups of 10 and 4 groups of 2.</p>  <p><math>4 \times 12 = 40 + 8</math></p>	<p>Understand how multiplication and partitioning are related through addition.</p>  <p><math>4 \times 3 = 12</math>    <math>4 \times 5 = 20</math>                      <math>4 \times 8 = 32</math></p> <p><math>4 \times 3 = 12</math>  <math>4 \times 5 = 20</math>  <math>12 + 20 = 32</math></p> <p><math>4 \times 8 = 32</math></p>	<p>Use partitioning to multiply 2-digit numbers by a single digit.</p> <p><math>18 \times 6 = ?</math></p>  <p><math>18 \times 6 = 10 \times 6 + 8 \times 6</math>  <math>= 60 + 48</math>  <math>= 108</math></p> <p><math>18 \times 6 = 10 \times 6 + 8 \times 6</math>  <math>= 60 + 48</math>  <math>= 108</math></p>



Column multiplication for 2- and 3-digit numbers multiplied by a single digit

Use place value equipment to make multiplications.

Make  $4 \times 136$  using equipment.



I can work out how many 1s, 10s and 100s.

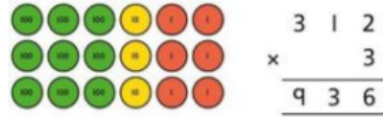
There are  $4 \times 6$  ones... 24 ones

There are  $4 \times 3$  tens ... 12 tens

There are  $4 \times 1$  hundreds ... 4 hundreds

$$24 + 120 + 400 = 544$$

Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.



Use the formal column method for up to 3-digit numbers multiplied by a single digit.

$$\begin{array}{r} 312 \\ \times 3 \\ \hline 936 \end{array}$$

Understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation.

$$\begin{array}{r} 23 \\ \times 5 \\ \hline 15 \\ 100 \\ \hline 115 \end{array} \qquad \begin{array}{r} 23 \\ \times 5 \\ \hline 115 \\ 1 \end{array}$$

3. Grid method (using place value counters or Dienes)

- TO x O

$$\begin{array}{|c|c|c|c|} \hline 1 & 8 & \times & 3 = 54 \\ \hline \times & 1 & 0 & 8 \\ \hline 3 & 3 & 0 & 24 \\ \hline \end{array}$$

The same layout is used as before but this time, the digits are being used.

- HTO x O

$$\begin{array}{|c|c|c|c|c|c|} \hline 1 & 3 & 5 & \times & 5 = & 675 \\ \hline \times & 1 & 0 & 0 & 3 & 0 \\ \hline 5 & 5 & 0 & 0 & 1 & 5 & 0 \\ \hline & & & & & & 25 \end{array}$$

The three digit number is partitioned horizontally with the hundreds first followed by the tens and ones.

Multiplying more than two numbers

Represent situations by multiplying three numbers together.



Each sheet has  $2 \times 5$  stickers.  
There are 3 sheets.

There are  $5 \times 2 \times 3$  stickers in total.

$$\begin{array}{l} 5 \times 2 \times 3 = 30 \\ \underbrace{\hspace{1.5cm}} \\ 10 \times 3 = 30 \end{array}$$

Understand that commutativity can be used to multiply in different orders.



$$\begin{array}{l} 2 \times 6 \times 10 = 120 \\ 12 \times 10 = 120 \end{array}$$

$$\begin{array}{l} 10 \times 6 \times 2 = 120 \\ 60 \times 2 = 120 \end{array}$$

Use knowledge of factors to simplify some multiplications.

$$24 \times 5 = 12 \times 2 \times 5$$

$$12 \times 2 \times 5 =$$



$$12 \times 10 = 120$$

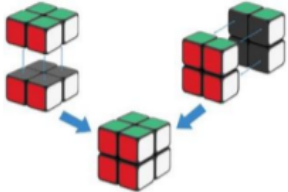
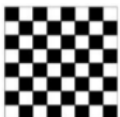
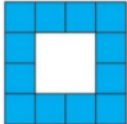


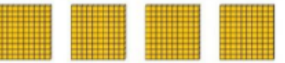


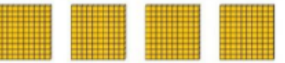



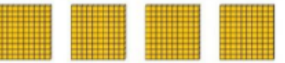
$$\text{So, } 24 \times 5 = 120$$

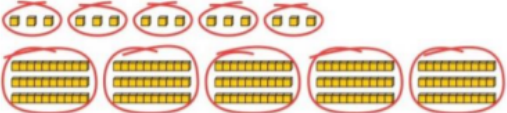

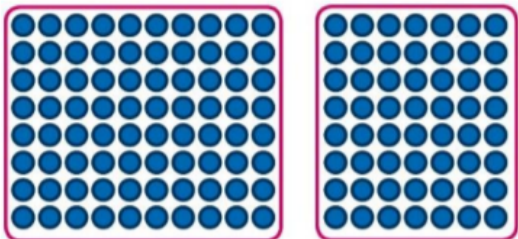
## Year 5 Multiplication

Pupils should be taught to:

- multiply numbers up to 4 digits by a one- or two digit number using a formal written method, including long multiplication for two-digit numbers
- multiply numbers mentally, drawing upon known facts
- multiply whole numbers and those involving decimals by 10,100 and 1,000
- solve problems involving multiplication, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving multiplication including understanding the meaning of the equals sign



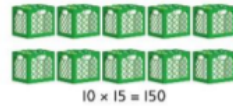
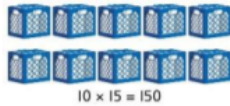
<p>Understanding factors</p>	<p>Use cubes or counters to explore the meaning of 'square numbers'.</p> <p><i>25 is a square number because it is made from 5 rows of 5.</i></p> <p>Use cubes to explore cube numbers.</p>  <p><i>8 is a cube number.</i></p>	<p>Use images to explore examples and non-examples of square numbers.</p>  <p><math>8 \times 8 = 64</math> <math>8^2 = 64</math></p>  <p><i>12 is not a square number, because you cannot multiply a whole number by itself to make 12.</i></p>	<p>Understand the pattern of square numbers in the multiplication tables.</p> <p>Use a multiplication grid to circle each square number. Can children spot a pattern?</p>												
<p>Multiplying by 10, 100 and 1,000</p>	<p>Use place value equipment to multiply by 10, 100 and 1,000 by unitising.</p> <table border="1" data-bbox="365 821 860 962"> <tr> <td><math>4 \times 1 = 4 \text{ ones} = 4</math></td> <td></td> </tr> <tr> <td><math>4 \times 10 = 4 \text{ tens} = 40</math></td> <td></td> </tr> <tr> <td><math>4 \times 100 = 4 \text{ hundreds} = 400</math></td> <td></td> </tr> </table>	$4 \times 1 = 4 \text{ ones} = 4$		$4 \times 10 = 4 \text{ tens} = 40$		$4 \times 100 = 4 \text{ hundreds} = 400$		<p>Understand the effect of repeated multiplication by 10.</p> 	<p>Understand how exchange relates to the digits when multiplying by 10, 100 and 1,000.</p> <table border="1" data-bbox="1541 863 1890 986"> <thead> <tr> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr> <td></td> <td>1</td> <td>7</td> </tr> </tbody> </table> <p><math>17 \times 10 = 170</math>  <math>17 \times 100 = 17 \times 10 \times 10 = 1,700</math>  <math>17 \times 1,000 = 17 \times 10 \times 10 \times 10 = 17,000</math></p>	H	T	O		1	7
$4 \times 1 = 4 \text{ ones} = 4$															
$4 \times 10 = 4 \text{ tens} = 40$															
$4 \times 100 = 4 \text{ hundreds} = 400$															
H	T	O													
	1	7													

<p>Multiplying by multiples of 10, 100 and 1,000</p>	<p>Use place value equipment to explore multiplying by unitising.</p>  <p>5 groups of 3 ones is 15 ones. 5 groups of 3 tens is 15 tens.</p> <p>So, I know that 5 groups of 3 thousands would be 15 thousands.</p>	<p>Use place value equipment to represent how to multiply by multiples of 10, 100 and 1,000.</p>  <p><math>4 \times 3 = 12</math> <math>4 \times 300 = 1,200</math></p> <p><math>6 \times 4 = 24</math> <math>6 \times 400 = 2,400</math></p>	<p>Use known facts and unitising to multiply.</p> <p><math>5 \times 4 = 20</math> <math>5 \times 40 = 200</math> <math>5 \times 400 = 2,000</math> <math>5 \times 4,000 = 20,000</math></p> <p><math>5,000 \times 4 = 20,000</math></p>																																								
<p>Multiplying up to 4-digit numbers by a single digit</p>	<p>Explore how to use partitioning to multiply efficiently.</p> <p><math>8 \times 17 = ?</math></p>  <p><math>8 \times 10 = 80</math> <math>8 \times 7 = 56</math></p> <p><math>80 + 56 = 136</math></p> <p>So, <math>8 \times 17 = 136</math></p>	<p>Represent multiplications using place value equipment and add the 1s, then 10s, then 100s, then 1,000s.</p> <table border="1" data-bbox="974 742 1400 1125"> <thead> <tr> <th></th> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr> <td>1000</td> <td></td> <td></td> <td>1 1 1</td> </tr> <tr> <td>100</td> <td></td> <td>10 10 10 10 10</td> <td>1 1 1</td> </tr> <tr> <td>100</td> <td></td> <td>10</td> <td></td> </tr> <tr> <td>100</td> <td></td> <td>10 10 10 10 10</td> <td>1 1 1</td> </tr> <tr> <td>100</td> <td></td> <td>10</td> <td></td> </tr> <tr> <td>100</td> <td></td> <td>10 10 10 10 10</td> <td>1 1 1</td> </tr> <tr> <td>100</td> <td></td> <td>10</td> <td></td> </tr> </tbody> </table>		H	T	O	1000			1 1 1	100		10 10 10 10 10	1 1 1	100		10		100		10 10 10 10 10	1 1 1	100		10		100		10 10 10 10 10	1 1 1	100		10		<p>Use an area model and then add the parts.</p> <table border="1" data-bbox="1545 702 2083 774"> <thead> <tr> <th></th> <th>100</th> <th>60</th> <th>3</th> </tr> </thead> <tbody> <tr> <td>5</td> <td><math>100 \times 5 = 500</math></td> <td><math>60 \times 5 = 300</math></td> <td><math>3 \times 5 = 15</math></td> </tr> </tbody> </table> <p>Use a column multiplication, including any required exchanges.</p> $\begin{array}{r} 136 \\ \times 6 \\ \hline 816 \\ \underline{23} \end{array}$		100	60	3	5	$100 \times 5 = 500$	$60 \times 5 = 300$	$3 \times 5 = 15$
	H	T	O																																								
1000			1 1 1																																								
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5	$100 \times 5 = 500$	$60 \times 5 = 300$	$3 \times 5 = 15$																																								

Multiplying 2-digit numbers by 2-digit numbers

Partition one number into 10s and 1s, then add the parts.

$$23 \times 15 = ?$$



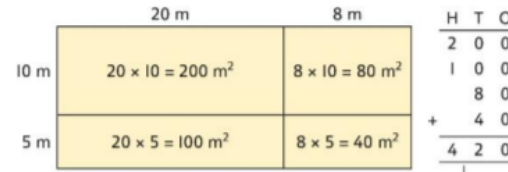
There are 345 bottles of milk in total.

H	T	O
1	5	0
1	5	0
+		4 5
3	4	5

$$23 \times 15 = 345$$

Use an area model and add the parts.

$$28 \times 15 = ?$$



$$28 \times 15 = 420$$

Use column multiplication, ensuring understanding of place value at each stage.

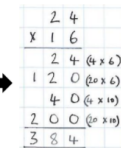
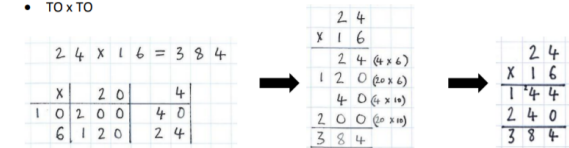
$$\begin{array}{r} 34 \\ \times 27 \\ \hline 238 \end{array} \quad 34 \times 7$$

$$\begin{array}{r} 34 \\ \times 27 \\ \hline 238 \\ 680 \end{array} \quad \begin{array}{l} 34 \times 7 \\ 34 \times 20 \end{array}$$

$$\begin{array}{r} 34 \\ \times 27 \\ \hline 238 \\ 680 \\ \hline 918 \end{array} \quad \begin{array}{l} 34 \times 7 \\ 34 \times 20 \\ 34 \times 27 \end{array}$$

5. Long Multiplication (introduced alongside grid method)

- TO x TO



Multiplying up to 4-digits by 2-digits

Use the area model then add the parts.

	100	40	3
10			
2			

	Th	H	T	O
	1	0	0	0
		4	0	0
		2	0	0
			8	0
			3	0
+				6
	1	7	1	6

$143 \times 12 = 1,716$   
There are 1,716 boxes of cereal in total.

$143 \times 12 = 1,716$

Use column multiplication, ensuring understanding of place value at each stage.

1 4 3	
x 1 2	
2 8 6	$143 \times 2$
1 4 3 0	$143 \times 10$
1 7 1 6	$143 \times 12$

Progress to include examples that require multiple exchanges as understanding, confidence and fluency build.

$1,274 \times 32 = ?$   
First multiply 1,274 by 2.

1 2 7 4	
x 3 2	
2 5 4 8	$1,274 \times 2$

Then multiply 1,274 by 30.

1 2 7 4	
x 3 2	
2 5 4 8	$1,274 \times 2$
3 8 2 2 0	$1,274 \times 30$

Finally, find the total.

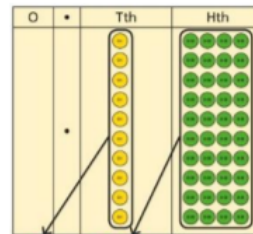
1 2 7 4	
x 3 2	
2 5 4 8	$1,274 \times 2$
3 8 2 2 0	$1,274 \times 30$
4 0 7 6 8	$1,274 \times 32$

$1,274 \times 32 = 40,768$

Multiplying decimals by 10, 100 and 1,000

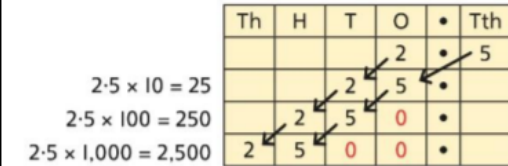
Use place value equipment to explore and understand the exchange of 10 tenths, 10 hundredths or 10 thousandths.

Represent multiplication by 10 as exchange on a place value grid.



$$0.14 \times 10 = 1.4$$

Understand how this exchange is represented on a place value chart.



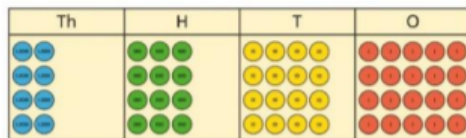
## Year 6 Multiplication

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operation to carry out calculations involving the 4 operations
- solve problems involving multiplication

Multiplying up to a 4-digit number by a single digit number

Use equipment to explore multiplications.



4 groups of 2,345

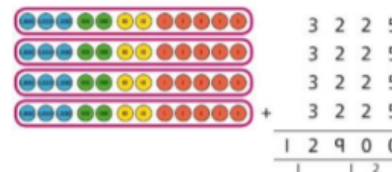
This is a multiplication:

$$4 \times 2,345$$

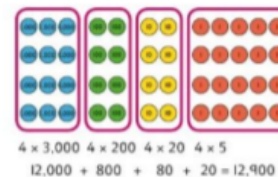
$$2,345 \times 4$$

Use place value equipment to compare methods.

Method 1



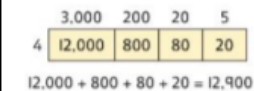
Method 2



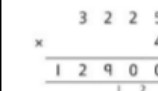
Understand area model and short multiplication.

Compare and select appropriate methods for specific multiplications.

Method 3



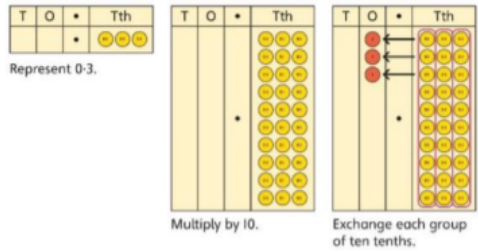
Method 4





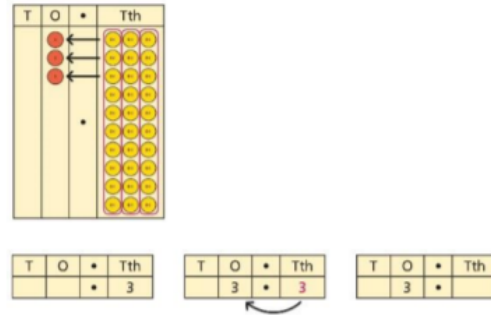
Multiplying by 10, 100 and 1,000

Use place value equipment to explore exchange in decimal multiplication.



$0.3 \times 10 = ?$   
 $0.3$  is 3 tenths.  
 $10 \times 3$  tenths are 30 tenths.  
 30 tenths are equivalent to 3 ones.

Understand how the exchange affects decimal numbers on a place value grid.



$0.3 \times 10 = 3$

Use knowledge of multiplying by 10, 100 and 1,000 to multiply by multiples of 10, 100 and 1,000.

$$8 \times 100 = 800$$

$$8 \times 300 = 800 \times 3 = 2,400$$

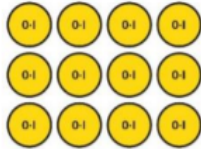
$$2.5 \times 10 = 25$$

$$2.5 \times 20 = 2.5 \times 10 \times 2 = 50$$

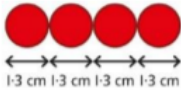


## Multiplying decimals

Explore decimal multiplications using place value equipment and in the context of measures.



3 groups of 4 tenths is 12 tenths.  
4 groups of 3 tenths is 12 tenths.



$4 \times 1 \text{ cm} = 4 \text{ cm}$   
 $4 \times 0.3 \text{ cm} = 1.2 \text{ cm}$   
 $4 \times 1.3 = 4 + 1.2 = 5.2 \text{ cm}$

Represent calculations on a place value grid.

$$3 \times 3 = 9$$

$$3 \times 0.3 = 0.9$$

T	O	•	Tth

Understand the link between multiplying decimals and repeated addition.



Use known facts to multiply decimals.

$$4 \times 3 = 12$$

$$4 \times 0.3 = 1.2$$

$$4 \times 0.03 = 0.12$$

$$20 \times 5 = 100$$

$$20 \times 0.5 = 10$$

$$20 \times 0.05 = 1$$

Find families of facts from a known multiplication.

*I know that  $18 \times 4 = 72$ .*

*This can help me work out:*

$$1.8 \times 4 = ?$$

$$18 \times 0.4 = ?$$

$$180 \times 0.4 = ?$$

$$18 \times 0.04 = ?$$

Use a place value grid to understand the effects of multiplying decimals.

	H	T	O	•	Tth	Hth
$2 \times 3$			6	•		
$0.2 \times 3$			0	•	6	
$0.02 \times 3$				•		



## Year 1 Division

Pupils should be taught to:

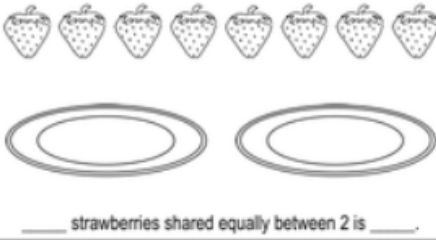
- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

Division as sharing

Practice sharing into equal groups.



Share the strawberries equally between the 2 plates and complete the sentence below.



\_\_\_ strawberries shared equally between 2 is \_\_\_.

Complete a division calculation

$$8 \div 2 = 4$$

## Year 2 Division

Pupils should be able to:

- recall and use division facts for the 2, 5 and 10
- calculate mathematical statements for multiplication and division
- write the division ( $\div$ ) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

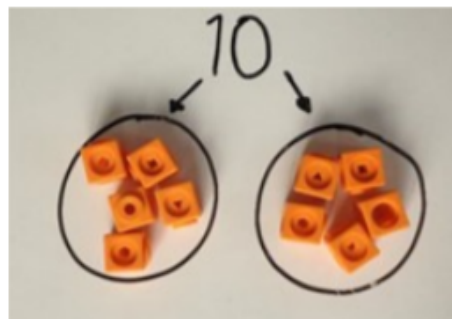
Objectives and strategies

Concrete

Pictorial

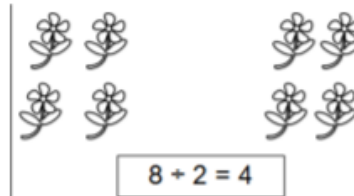
Abstract

Division as sharing



I have 10 cubes, can you share them equally in 2 groups?

Children use pictures or shapes to share quantities.



Draw sharing model to support division and halving.

Half of 10 is -

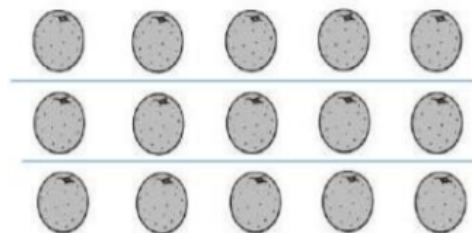
$$28 \div 7 = 4$$

Divide 28 into 7 groups. How many are in each group?

Division as grouping

Division by grouping-  
Group objects into their divisible factor.

Draw an array and use lines to split the array into groups to make multiplication and division sentences.



Find the inverse of multiplication and division sentences by creating four linking number sentences.

$$7 \times 4 = 28$$

$$4 \times 7 = 28$$

$$28 \div 7 = 4$$

$$28 \div 4 = 7$$

## Division within arrays

Link division to multiplication by creating an array and thinking about the number sentences that can be created.



Eg.

$$15 \div 3 = 5$$

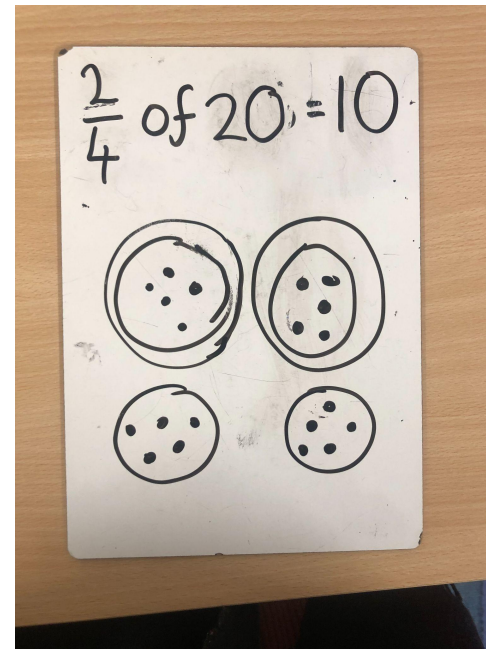
$$5 \times 3 = 15$$

$$15 \div 5 = 3$$

$$3 \times 5 = 15$$

## Division in quarters

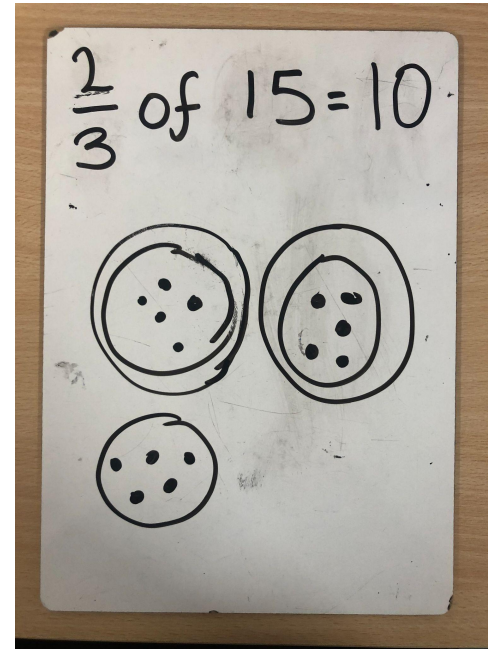
Dividing objects into 4 groups and counting the fraction of the amount.



$$\frac{2}{4} \text{ of } 20 = 10$$

Division in thirds

Dividing objects into 3 groups and counting the fraction of the amount.



$\frac{2}{3}$  of 15 = 10

## Year 3 Division

Pupils should be taught to:

- recall and use division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving division, including positive integer scaling problems and correspondence problems in which  $n$  objects are connected to  $m$  objects

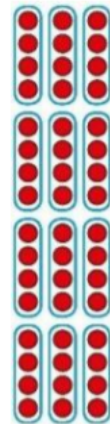
Using times tables knowledge to divide

Use knowledge of known times-tables to calculate divisions.



24 divided into groups of 8.  
There are 3 groups of 8.

Use knowledge of known times-tables to calculate divisions.



$$48 \div 4 = 12$$

48 divided into groups of 4.  
There are 12 groups.

$$4 \times 12 = 48$$

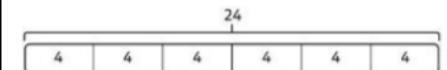
$$48 \div 4 = 12$$

Use knowledge of known times-tables to calculate divisions.

*I need to work out 30 shared between 5.*

*I know that  $6 \times 5 = 30$   
so I know that  $30 \div 5 = 6$ .*

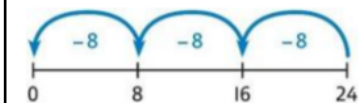
A bar model may represent the relationship between sharing and grouping.



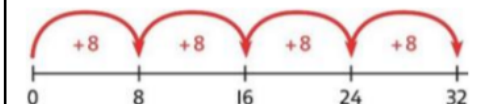
$$24 \div 4 = 6$$

$$24 \div 6 = 4$$


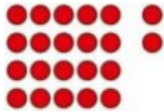


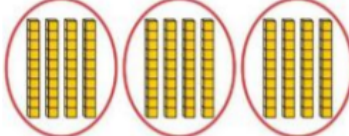
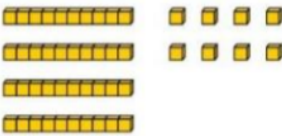
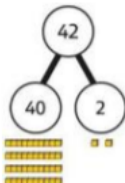
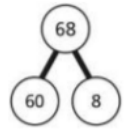
Children understand how division is related to both repeated subtraction and repeated addition.

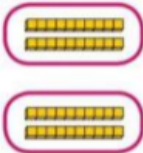

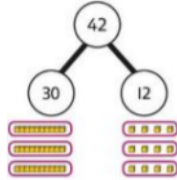




$$24 \div 8 = 3$$



$$32 \div 8 = 4$$

<p>Understanding remainders</p>	<p>Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further.</p>  <p><i>There are 13 sticks in total. There are 3 groups of 4, with 1 remainder.</i></p>	<p>Use images to explain remainders.</p>  <p><math>22 \div 5 = 4 \text{ remainder } 2</math></p>	<p>Understand that the remainder is what cannot be shared equally from a set.</p> <p><math>22 \div 5 = ?</math></p> <p><math>3 \times 5 = 15</math>  <math>4 \times 5 = 20</math>  <math>5 \times 5 = 25 \dots \text{this is larger than } 22</math>      So, <math>22 \div 5 = 4 \text{ remainder } 2</math></p>
<p>Using known facts to divide multiples of 10</p>	<p>Use place value equipment to understand how to divide by unitising.</p> <p><i>Make 6 ones divided by 3.</i></p>  <p><i>Now make 6 tens divided by 3.</i></p>  <p><i>What is the same? What is different?</i></p>	<p>Divide multiples of 10 by unitising.</p>  <p><i>12 tens shared into 3 equal groups. 4 tens in each group.</i></p>	<p>Divide multiples of 10 by a single digit using known times-tables.</p> <p><math>180 \div 3 = ?</math></p> <p><i>180 is 18 tens.</i></p> <p><i>18 divided by 3 is 6. 18 tens divided by 3 is 6 tens.</i></p> <p><math>18 \div 3 = 6</math>  <math>180 \div 3 = 60</math></p>
<p>2-digit number divided by 1-digit number, no remainders</p>	<p>Children explore dividing 2-digit numbers by using place value equipment.</p>  <p><math>48 \div 2 = ?</math></p>	<p>Children explore which partitions support particular divisions.</p> 	<p>Children partition a number into 10s and 1s to divide where appropriate.</p>  <p><math>60 \div 2 = 30</math>  <math>8 \div 2 = 4</math>  <math>30 + 4 = 34</math>  <math>68 \div 2 = 34</math></p>


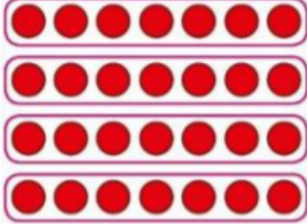
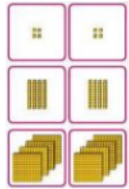



	<p><i>First divide the 10s.</i></p>  <p><i>Then divide the 1s.</i></p> 	<p><i>I need to partition 42 differently to divide by 3.</i></p>  <p><math>42 = 30 + 12</math></p> <p><math>42 \div 3 = 14</math></p>	<p>Children partition flexibly to divide where appropriate.</p> <p><math>42 \div 3 = ?</math>  <math>42 = 40 + 2</math></p> <p><i>I need to partition 42 differently to divide by 3.</i></p> <p><math>42 = 30 + 12</math></p> <p><math>30 \div 3 = 10</math>  <math>12 \div 3 = 4</math></p> <p><math>10 + 4 = 14</math>  <math>42 \div 3 = 14</math></p>
<p>2-digit number divided by 1-digit number, with remainders</p>	<p>Use place value equipment to understand the concept of remainder.</p> <p><i>Make 29 from place value equipment. Share it into 2 equal groups.</i></p>  <p><i>There are two groups of 14 and 1 remainder.</i></p>	<p>Use place value equipment to understand the concept of remainder in division.</p> <p><math>29 \div 2 = ?</math></p>  <p><math>29 \div 2 = 14 \text{ remainder } 1</math></p>	<p>Partition to divide, understanding the remainder in context.</p> <p><i>67 children try to make 5 equal lines.</i></p> <p><math>67 = 50 + 17</math>  <math>50 \div 5 = 10</math></p> <p><math>17 \div 5 = 3 \text{ remainder } 2</math>  <math>67 \div 5 = 13 \text{ remainder } 2</math></p> <p><i>There are 13 children in each line and 2 children left out.</i></p>

## Year 4 Division

Pupils should be taught to:

- recall division facts for multiplication tables up to 12 x 12
- use place value, known and derived facts to divide mentally
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects



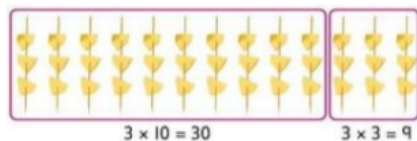
<p>Understanding the relationship between multiplication and division, including times-tables</p>	<p>Use objects to explore families of multiplication and division facts.</p>  <p><math>4 \times 6 = 24</math>  <i>24 is 6 groups of 4.</i>  <i>24 is 4 groups of 6.</i></p> <p><i>24 divided by 6 is 4.</i>  <i>24 divided by 4 is 6.</i></p>	<p>Represent divisions using an array.</p>  <p><math>28 \div 7 = 4</math></p>	<p>Understand families of related multiplication and division facts.</p> <p><i>I know that <math>5 \times 7 = 35</math></i></p> <p><i>so I know all these facts:</i></p> <p><math>5 \times 7 = 35</math>  <math>7 \times 5 = 35</math>  <math>35 = 5 \times 7</math>  <math>35 = 7 \times 5</math>  <math>35 \div 5 = 7</math>  <math>35 \div 7 = 5</math>  <math>7 = 35 \div 5</math>  <math>5 = 35 \div 7</math></p>
<p>Dividing multiples of 10 and 100 by a single digit</p>	<p>Use place value equipment to understand how to use unitising to divide.</p>  <p><i>8 ones divided into 2 equal groups</i>  <i>4 ones in each group</i></p> <p><i>8 tens divided into 2 equal groups</i>  <i>4 tens in each group</i></p> <p><i>8 hundreds divided into 2 equal groups</i>  <i>4 hundreds in each group</i></p>	<p>Represent divisions using place value equipment.</p> <p><math>9 \div 3 = \square</math></p>  <p><math>90 \div 3 = \square</math></p>  <p><math>900 \div 3 = \square</math></p>  <p><math>9 \div 3 = 3</math></p> <p><i>9 tens divided by 3 is 3 tens.</i>  <i>9 hundreds divided by 3 is 3 hundreds.</i></p>	<p>Use known facts to divide 10s and 100s by a single digit.</p> <p><math>15 \div 3 = 5</math></p> <p><math>150 \div 3 = 50</math></p> <p><math>1500 \div 3 = 500</math></p>



Dividing 2-digit and 3-digit numbers by a single digit by partitioning into 100s, 10s and 1s

Partition into 10s and 1s to divide where appropriate.

$$39 \div 3 = ?$$



$$39 = 30 + 9$$

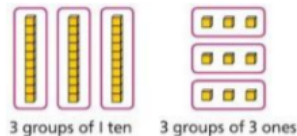
$$30 \div 3 = 10$$

$$9 \div 3 = 3$$

$$39 \div 3 = 13$$

Partition into 100s, 10s and 1s using Base 10 equipment to divide where appropriate.

$$39 \div 3 = ?$$



$$39 = 30 + 9$$

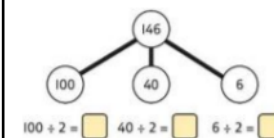
$$30 \div 3 = 10$$

$$9 \div 3 = 3$$

$$39 \div 3 = 13$$

Partition into 100s, 10s and 1s using a part-whole model to divide where appropriate.

$$142 \div 2 = ?$$



$$100 \div 2 = 50$$

$$40 \div 2 = 20$$

$$6 \div 2 = 3$$

$$50 + 20 + 3 = 73$$

$$142 \div 2 = 73$$

#### 6. TO ÷ O Formal method with chunking

- TO ÷ O (with no remainders)

$$96 \div 4 = 24$$

2	4
4	96
-	40
5	6
-	40
1	6
-	16
0	0

The chunking method is introduced but only with a single digit divisor. The number of groups should be recorded alongside on the right with the answer written on top of the bus stop.

- TO ÷ O (with remainders)

$$43 \div 3 = 14 \text{ r } 1$$

1	4	r	1
3	4	3	
-	3	0	
1	3		
-	1	2	
0	1		



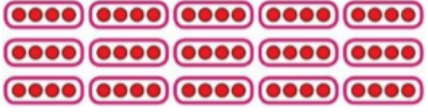

The same layout is then used again but with remainders.

## Year 5 Division

Pupils should be taught to:

- divide numbers mentally, drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- divide whole numbers and those involving decimals by 10, 100 and 1,000

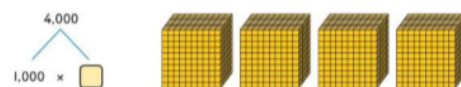
- solve problems involving division, including using their knowledge of factors and multiples squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving division, including scaling by simple fractions and problems involving simple rates

<p>Understanding factors and prime numbers</p>	<p>Use equipment to explore the factors of a given number.</p>  <p><math>24 \div 3 = 8</math>  <math>24 \div 8 = 3</math>  <i>8 and 3 are factors of 24 because they divide 24 exactly.</i></p> <p><math>24 \div 5 = 4</math> remainder 4.</p>  <p><i>5 is not a factor of 24 because there is a remainder.</i></p>	<p>Understand that prime numbers are numbers with exactly two factors.</p> <p><math>13 \div 1 = 13</math>  <math>13 \div 2 = 6</math> r 1  <math>13 \div 4 = 4</math> r 1</p> <p><i>1 and 13 are the only factors of 13. 13 is a prime number.</i></p>	<p>Understand how to recognise prime and composite numbers.</p> <p><i>I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder.</i></p> <p><i>I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33.</i></p> <p><i>I know that 1 is not a prime number, as it has only 1 factor.</i></p>
<p>Understanding inverse operations and the link with multiplication, grouping and sharing</p>	<p>Use equipment to group and share and to explore the calculations that are present.</p> <p><i>I have 28 counters.</i></p> <p><i>I made 7 groups of 4. There are 28 in total.</i></p> <p><i>I have 28 in total. I shared them equally into 7 groups. There are 4 in each group.</i></p> <p><i>I have 28 in total. I made groups of 4. There are 7 equal groups.</i></p>	<p>Represent multiplicative relationships and explore the families of division facts.</p>  <p><math>60 \div 4 = 15</math>  <math>60 \div 15 = 4</math></p>	<p>Represent the different multiplicative relationships to solve problems requiring inverse operations.</p> <p><math>12 \div 3 = \square</math>  <math>12 \div \square = 3</math>  <math>\square \times 3 = 12</math>  <math>\square \div 3 = 12</math></p>  <p>Understand missing number problems for division calculations and know how to solve them using inverse operations.</p> <p><math>22 \div ? = 2</math>  <math>22 \div 2 = ?</math>  <math>? \div 2 = 22</math>  <math>? \div 22 = 2</math></p>

## Dividing whole numbers by 10, 100 and 1,000

Use place value equipment to support unitising for division.

$$4,000 \div 1,000$$



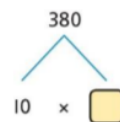
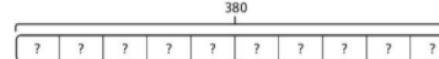
4,000 is 4 thousands.

$$4 \times 1,000 = 4,000$$

$$\text{So, } 4,000 \div 1,000 = 4$$

Use a bar model to support dividing by unitising.

$$380 \div 10 = 38$$



380 is 38 tens.

$$38 \times 10 = 380$$

$$10 \times 38 = 380$$

$$\text{So, } 380 \div 10 = 38$$

Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000.

Th	H	T	O
3	2	0	0

$$3,200 \div 100 = ?$$

3,200 is 3 thousands and 2 hundreds.

$$200 \div 100 = 2$$

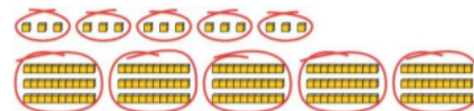
$$3,000 \div 100 = 30$$

$$3,200 \div 100 = 32$$

So, the digits will move two places to the right.

## Dividing by multiples of 10, 100 and 1,000

Use place value equipment to represent known facts and unitising.



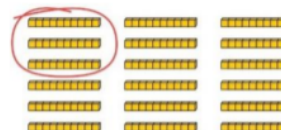
15 ones put into groups of 3 ones. There are 5 groups.

$$15 \div 3 = 5$$

15 tens put into groups of 3 tens. There are 5 groups.

$$150 \div 30 = 5$$

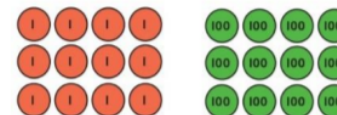
Represent related facts with place value equipment when dividing by unitising.



180 is 18 tens.

18 tens divided into groups of 3 tens. There are 6 groups.

$$180 \div 30 = 6$$



12 ones divided into groups of 4. There are 3 groups.

12 hundreds divided into groups of 4 hundreds. There are 3 groups.

$$1200 \div 400 = 3$$

Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check.

$$3,000 \div 5 = 600$$

$$3,000 \div 50 = 60$$

$$3,000 \div 500 = 6$$

$$5 \times 600 = 3,000$$

$$50 \times 60 = 3,000$$

$$500 \times 6 = 3,000$$

## Dividing up to four digits by a single digit using short division

Explore grouping using place value equipment.

$$268 \div 2 = ?$$

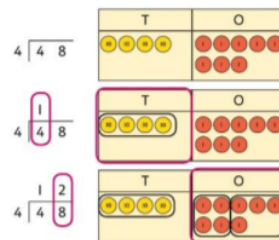
*There is 1 group of 2 hundreds.  
There are 3 groups of 2 tens.  
There are 4 groups of 2 ones.*

$$264 \div 2 = 134$$

Use place value equipment on a place value grid alongside short division.

The model uses grouping.

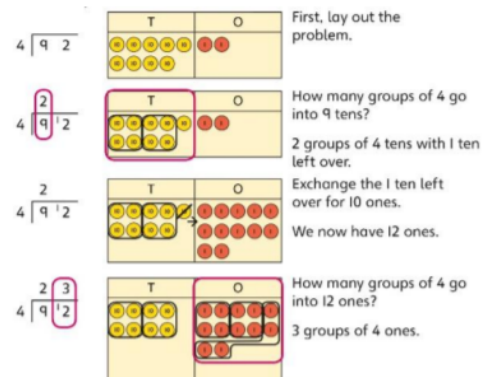
A sharing model can also be used, although the model would need adapting.



Lay out the problem as a short division.

*There is 1 group of 4 in 4 tens.  
There are 2 groups of 4 in 8 ones.*

Work with divisions that require exchange.



Use short division for up to 4-digit numbers divided by a single digit.

$$\begin{array}{r} 0 \ 5 \ 5 \ 6 \\ 7 \overline{) 3 \ 8 \ 9 \ 2} \end{array}$$

$$3,892 \div 7 = 556$$

Use multiplication to check.

$$556 \times 7 = ?$$

$$6 \times 7 = 42$$

$$50 \times 7 = 350$$

$$500 \times 7 = 3500$$

$$3,500 + 350 + 42 = 3,892$$

## Understanding remainders

Understand remainders using concrete versions of a problem.

80 cakes divided into trays of 6.



80 cakes in total. They make 13 groups of 6, with 2 remaining.

Use short division and understand remainders as the last remaining 1s.

6  $\overline{)80}$

T	O
●●●●●●	

Layout the problem as short division.

6  $\overline{)130}$

T	O
●●●●●●	

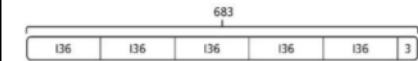
How many groups of 6 go into 8 tens?  
There is 1 group of 6 tens.  
There are 2 tens remaining.

6  $\overline{)130}$

T	O
●●●●●●	●●●●●●

How many groups of 6 go into 20 ones?  
There are 3 groups of 6 ones.  
There are 2 ones remaining.

In problem solving contexts, represent divisions including remainders with a bar model.



$$683 = 136 \times 5 + 3$$

$$683 \div 5 = 136 \text{ r } 3$$

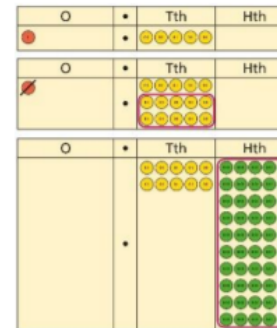
## Dividing decimals by 10, 100 and 1,000

Understand division by 10 using exchange.

2 ones are 20 tenths.

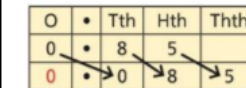
20 tenths divided by 10 is 2 tenths.

Represent division using exchange on a place value grid.

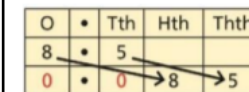


1·5 is 1 one and 5 tenths.  
This is equivalent to 10 tenths and 50 hundredths.  
10 tenths divided by 10 is 1 tenth.  
50 hundredths divided by 10 is 5 hundredths.  
1·5 divided by 10 is 1 tenth and 5 hundredths.  
 $1 \cdot 5 \div 10 = 0 \cdot 15$

Understand the movement of digits on a place value grid.



$$0 \cdot 85 \div 10 = 0 \cdot 085$$



$$8 \cdot 5 \div 100 = 0 \cdot 085$$

## Understanding the relationship between fractions and division

Use sharing to explore the link between fractions and division.

1 whole shared between 3 people.  
Each person receives one-third.



Use a bar model and other fraction representations to show the link between fractions and division.



$$1 \div 3 = \frac{1}{3}$$

Use the link between division and fractions to calculate divisions.

$$5 \div 4 = \frac{5}{4} = 1 \frac{1}{4}$$

$$11 \div 4 = \frac{11}{4} = 2 \frac{3}{4}$$

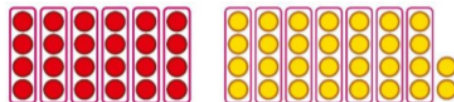
## Year 6 Division

Pupils should be taught to:

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operations to carry out calculations involving the 4 operations

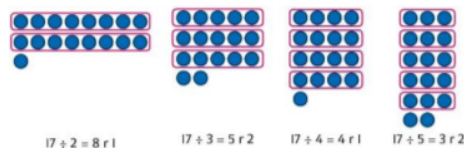
## Understanding factors

Use equipment to explore different factors of a number.



4 is a factor of 24 but is not a factor of 30.

Recognise prime numbers as numbers having exactly two factors. Understand the link with division and remainders.



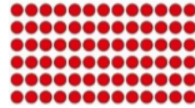
Recognise and know primes up to 100. Understand that 2 is the only even prime, and that 1 is not a prime number.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

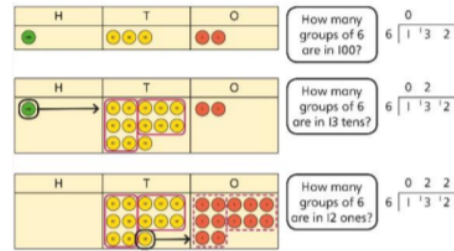


## Dividing by a single digit

Use equipment to make groups from a total.



There are 78 in total.  
There are 6 groups of 13.  
There are 13 groups of 6.



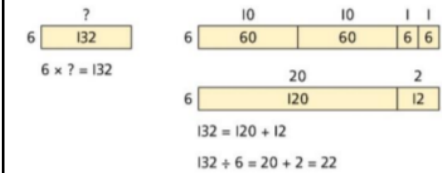
Use short division to divide by a single digit.

$$6 \overline{) 132}$$

$$6 \overline{) 132}$$

$$6 \overline{) 132}$$

Use an area model to link multiplication and division.



## Dividing by a 2-digit number using factors

Understand that division by factors can be used when dividing by a number that is not prime.

Use factors and repeated division.

$$1,260 \div 14 = ?$$



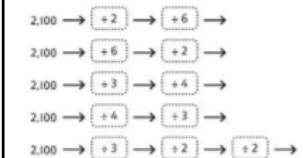
$$1,260 \div 2 = 630$$

$$630 \div 7 = 90$$

$$1,260 \div 14 = 90$$

Use factors and repeated division where appropriate.

$$2,100 \div 12 = ?$$



## Dividing by a 2-digit number using long division

Use equipment to build numbers from groups.



182 divided into groups of 13.  
There are 14 groups.

Use an area model alongside written division to model the process.

$$377 \div 13 = ?$$

$$\begin{array}{r} 13 \overline{) 377} \\ \underline{130} \phantom{0} \\ 247 \phantom{0} \\ \underline{130} \phantom{0} \\ 117 \phantom{0} \\ \underline{117} \\ 0 \end{array}$$

$$\begin{array}{r} 13 \overline{) 377} \\ \underline{130} \phantom{0} \\ 247 \phantom{0} \\ \underline{130} \phantom{0} \\ 117 \phantom{0} \\ \underline{117} \\ 0 \end{array}$$

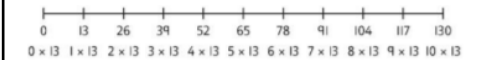
$$\begin{array}{r} 13 \overline{) 377} \\ \underline{130} \phantom{0} \\ 247 \phantom{0} \\ \underline{130} \phantom{0} \\ 117 \phantom{0} \\ \underline{117} \\ 0 \end{array}$$

$$\begin{array}{r} 13 \overline{) 377} \\ \underline{130} \phantom{0} \\ 247 \phantom{0} \\ \underline{130} \phantom{0} \\ 117 \phantom{0} \\ \underline{117} \\ 0 \end{array}$$

$$377 \div 13 = 29$$

Use long division where factors are not useful (for example, when dividing by a 2-digit prime number). Write the required multiples to support the division process.

$$377 \div 13 = ?$$



$$\begin{array}{r} 13 \overline{) 377} \\ - 130 \phantom{0} \\ \hline 247 \phantom{0} \\ - 130 \phantom{0} \\ \hline 117 \phantom{0} \\ - 117 \phantom{0} \\ \hline 0 \phantom{0} \end{array}$$

$$377 \div 13 = 29$$

A slightly different layout may be used, with the division completed above rather than at the side.

$$\begin{array}{r} 3 \\ 21 \overline{) 798} \\ - 630 \\ \hline 168 \\ - 168 \\ \hline 0 \end{array}$$

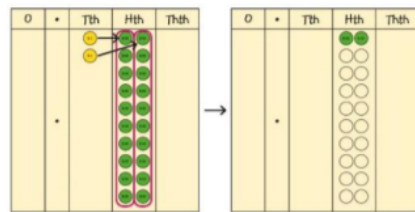
$$\begin{array}{r} 38 \\ 21 \overline{) 798} \\ - 630 \\ \hline 168 \\ - 168 \\ \hline 0 \end{array}$$

Divisions with a remainder explored in problem-solving contexts.



## Dividing by 10, 100 and 1,000

Use place value equipment to explore division as exchange.



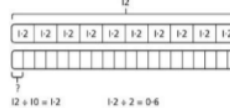
$0.2$  is 2 tenths.  
 2 tenths is equivalent to 20 hundredths.  
 20 hundredths divided by 10 is 2 hundredths.

Represent division to show the relationship with multiplication. Understand the effect of dividing by 10, 100 and 1,000 on the digits on a place value grid.



Understand how to divide using division by 10, 100 and 1,000.

$$12 \div 20 = ?$$



### 8. Chunking ( $\div 2$ digits)

- HTO  $\div$  TO (without remainders)

$$432 \div 16 = 27$$

```

    27
  16 432
    -160 (10 x 16)
    ---
     272
     -160 (10 x 16)
     ---
      80 (5 x 16)
      -80
      ---
       00
  
```

The formal chunking method is reintroduced with a two digit divisor.

- HTO  $\div$  TO (with remainders)

$$327 \div 19$$

```

    17 r 4
  19 327
    -190 (10 x 19)
    ---
     95 (5 x 19)
     -95
     ---
      04
  
```

Use knowledge of factors to divide by multiples of 10, 100 and 1,000.

$$40 \div 50 = \square$$

$$40 \rightarrow \boxed{\div 10} \rightarrow \boxed{\div 5} \rightarrow ?$$

$$40 \rightarrow \boxed{\div 5} \rightarrow \boxed{\div 10} \rightarrow ?$$

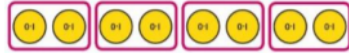
$$40 \div 5 = 8$$

$$8 \div 10 = 0.8$$

So,  $40 \div 50 = 0.8$

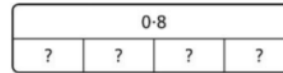
## Dividing decimals

Use place value equipment to explore division of decimals.



*8 tenths divided into 4 groups. 2 tenths in each group.*

Use a bar model to represent divisions.



$$4 \times 2 = 8$$

$$8 \div 4 = 2$$

So,  $4 \times 0.2 = 0.8$

$0.8 \div 4 = 0.2$

Use short division to divide decimals with up to 2 decimal places.

$$8 \overline{) 4.24}$$

0.

$$8 \overline{) 4.24}$$

0.5

$$8 \overline{) 4.24}$$

0.53

$$8 \overline{) 4.24}$$

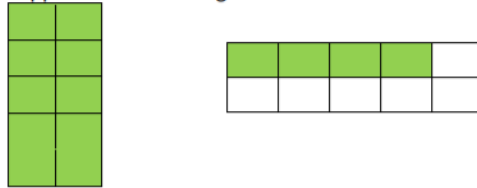
## Year 3 Fractions

### Pupils should be taught to:

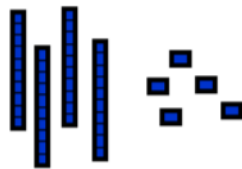
- count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit number or quantities by 10.
- recognise, find and write fractions of a discrete set of objects, unit fractions and non-unit fractions with small denominators
- recognise and use fractions as numbers, unit fractions and non-unit fractions with small denominators
- recognise and show, using diagrams, equivalent fractions with small denominators
- add and subtract fractions with the same denominator within one whole for example,  $5/7 + 1/7 = 6/7$
- compare and order unit fractions, and fractions with the same denominators
- solve problems that involve all of the above.

### Counting in tenths

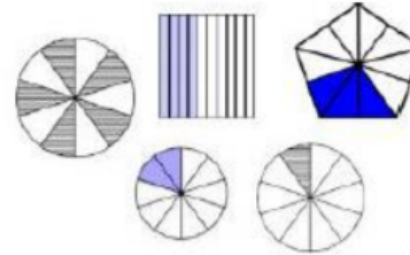
Children could use multiple tens frames to support them counting in tenths.



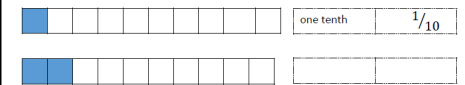
The children could use counters or shading. It is important that they can use the tens frame in different orientations.

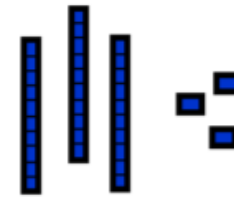
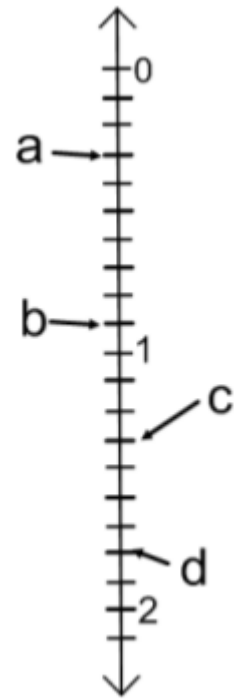


To start with, use the tenths cubes to embed the idea that 10 tenths = 1 whole. Children can then exchange their ten tenths for a 1 whole stick. Start at different points to rehearse counting forwards and backwards.



What does each image represent?  
How many more sections would I need to shade to make  $2\frac{7}{10}$ ?





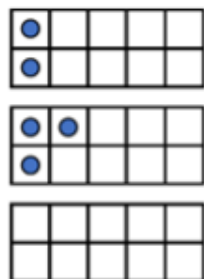
Sam started counting at  $2\frac{7}{10}$ . He finished with the picture above. How many tenths has he counted on?  
 Ext: This is Sam's finished image. Where could he have started counting? How many tenths away from this is it?

$2\frac{6}{10}$	$2\frac{7}{10}$			
	$3\frac{7}{10}$		$3\frac{9}{10}$	
		$4\frac{8}{10}$		

Here is part of a number square. Fill in the missing numbers.  
 (N.B. to extend more able learners, you may wish to remove  $3\frac{9}{10}$  or extend the table beyond a whole.)



Recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10

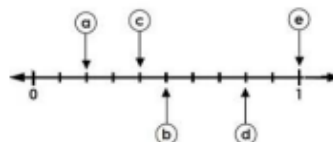
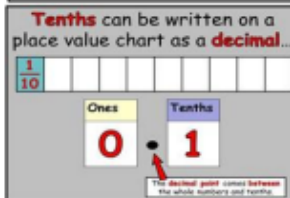
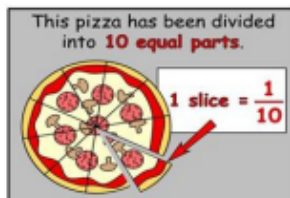
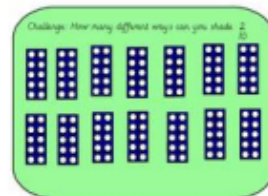


Use tens frames to represent tenths and count in tenths. Could also use a ten piece from numicon set with an object into the circles to represent the amount of tenths.

Using 10p coins with 10 adding up to £1 - also links to the decimal place.



How many tenths of a whole pound do you have? 3/10 link to how it would be written as money £0.30.



Which tenth is represented by the letter a? B? C? D? E?

Eight tenths  $\frac{7}{10}$

  $\frac{8}{10}$

  $\frac{6}{10}$

**What do you notice?**

$$\frac{1}{10} \text{ of } 10 = 1$$

$$\frac{2}{10} \text{ of } 10 = 2$$

$$\frac{3}{10} \text{ of } 10 = 3$$

Continue the pattern.

**What do you notice?**

What about  $\frac{1}{10}$  of 20? Use this to work out  $\frac{2}{10}$  of 20, etc.

$$\frac{1}{10} \text{ of } 100 = 10$$

$$\frac{1}{100} \text{ of } 100 = 1$$

$$\frac{2}{10} \text{ of } 100 = 20$$

$$\frac{2}{100} \text{ of } 100 = 2$$

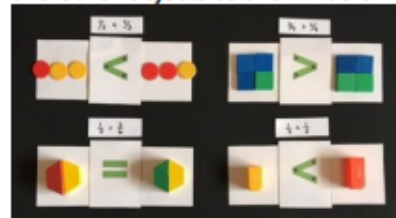
How can you use this to work out  $\frac{6}{10}$  of 200?  $\frac{6}{100}$  of 200?



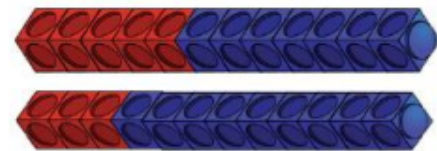
Compare and order unit fractions, and fractions with the same denominators

Equipment that could be used:  
Paper strips; Counters; Cubes; Fraction tiles; Fraction rods; Cuisenaire rods

E.g. Compare fractions using counters and other objects as shown below

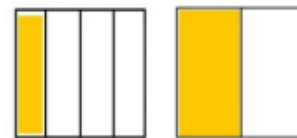


Paper strips can be used to help compare fractions. Ensure that the paper strips are of equal size.



Use cube towers to compare fractions.

This could begin by using paper strips before exposing children to pictorial representations such as:



$$\frac{1}{4}$$

$$\frac{1}{2}$$



$$\frac{3}{8}$$

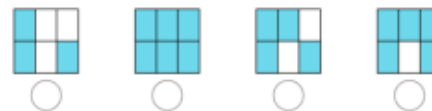


$$\frac{5}{8}$$



$$\frac{7}{8}$$

Order the fractions in each row from smallest to largest.  
Use 1 for the smallest and 4 for the largest fraction.



For example:

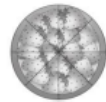
Compare using =, < or >  
 $\frac{1}{2}$ ,  $\frac{1}{5}$ ,  $\frac{1}{3}$ ,  $\frac{3}{4}$

Compare using =, < or >  
 $\frac{2}{7}$ ,  $\frac{5}{7}$ ,  $\frac{4}{8}$ ,  $\frac{5}{8}$

$$\begin{matrix} \boxed{1} & \boxed{2} & \boxed{3} & \boxed{4} & \boxed{4} \\ \frac{1}{4} & < & \frac{B}{C} \end{matrix}$$

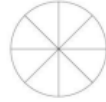
Order the following fractions from smallest to largest:

## Add and subtract fractions with the same denominator within one whole



Provide pupils with a strawberry tart cut into eighths and an identically sized and cut blank copy.

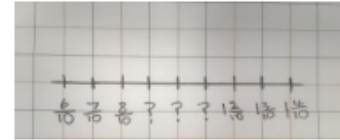
*Collins Shanghai Y3 Unit 8.4*



Determine that each part represents one eighth of the tart because the whole has been divided into eight equal parts.

Get the children to cut out each part of the pie and label them as  $\frac{1}{8}$ . Hold up one piece in each hand and elicit that this is  $\frac{2}{8}$ . Record the calculation:  $\frac{1}{8} + \frac{1}{8} = \frac{2}{8}$ . Relate the common denominators to the number of equal pieces of the tart, and then discuss how by adding two of them together they get  $\frac{2}{8}$ . Ask what would happen if one more eighth was added to the new strawberry tart. Stick another eighth on to get  $\frac{3}{8}$ . Continue this process. Put the final piece on and remind the children that  $\frac{8}{8}$  is the same as one whole (strawberry tart).

Count up and down in fraction amounts on a number line.

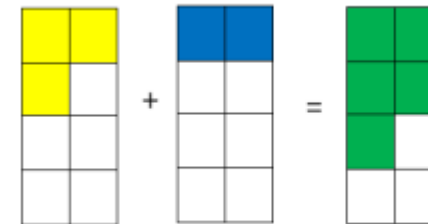


*Twinkl*

Count up in fraction amounts using paper cards.



*I See*



Make sure the numerators are the same, then add the denominators.

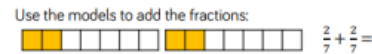
$$\text{e.g. } \frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$





When using numicon, the base piece represents the denominator and the top pieces represent the numerator. Cubes or pegs could also be used to represent the numerators.

Cube towers can be used:



Similar resources can also be used to demonstrate subtraction of fractions.



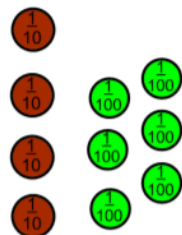
## Year 4 Fractions

Pupils should be taught to:

- recognise and show, using diagrams, families of common equivalent fractions
- count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- add and subtract fractions with the same denominator
- recognise and write decimal equivalents of any number of tenths or hundredths
- recognise and write decimal equivalents to  $1/4$ ,  $1/2$ ,  $3/4$
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths
- round decimals with one decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to two decimal places
- solve simple measure and money problems involving fractions and decimals to two decimal places.

Counting up and down in hundredths.

Use 100 bead strings to rehearse counting forwards and backwards. Use different starting points and draw discussion into the fact that  $\frac{1}{10} = \frac{10}{100}$ .



To start with, use the hundredths counters to embed the idea that 10 hundredths = 1 tenth. Children can then exchange their ten hundredths for a tenth counter. Start at different points to rehearse counting forwards and backwards.

As for counting in tenths  
Use dienes units blocks to represent  $\frac{1}{100}$   
Use pennies to recognise  $\frac{1}{100}$  of a pound etc

Decimal Number Chart 0.01-1

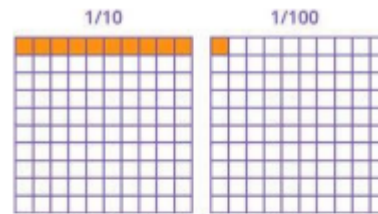
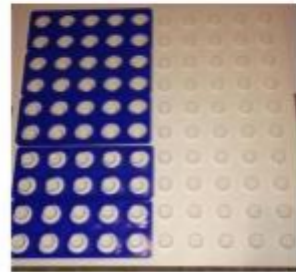
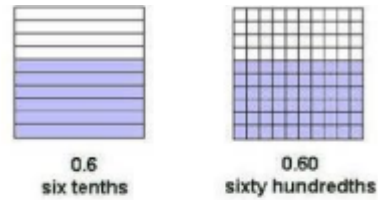
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.2
0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.3
0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.4
0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.5
0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59	0.6
0.61	0.62	0.63	0.64	0.65	0.66	0.67	0.68	0.69	0.7
0.71	0.72	0.73	0.74	0.75	0.76	0.77	0.78	0.79	0.8
0.81	0.82	0.83	0.84	0.85	0.86	0.87	0.88	0.89	0.9
0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1

As for counting in tenths  
9.8 is equivalent to 980 hundredths,  
9.82 is equivalent to ? hundredths.  
How many hundredths are there in 10.0?

Using a partially filled in hundredths square, ask how you know where e.g. 2.76 would go.

2.5	2.5				
2	3				
					2.6
					7
2.8			2.8	2.8	
2			5	6	
				2.9	
				6	

Recognise and write decimal equivalents of any number of tenths or hundredths



$\frac{60}{100}$	0.60	six tenths	
$\frac{70}{100}$	0.70	seven tenths	
$\frac{8}{10}$	0.8	eighty hundredths	

**What do you notice?**

One tenth of £41

One hundredth of £41

One thousandth of £41 Continue the pattern. What do you notice?

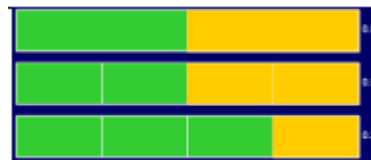
$$0.085 + 0.015 = 0.1$$

$$0.075 + 0.025 = 0.1$$

$$0.065 + 0.035 = 0.1$$

Continue the pattern for the next five number sentences

Recognise and write decimal equivalents to  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{3}{4}$



Explore using link:

<https://mathsframe.co.uk/en/resources/resource/66/itp-fractions>



Match each fraction to its decimal equivalent.

$\frac{1}{2}$        $\frac{4}{10}$        $\frac{3}{4}$        $\frac{1}{4}$   
 0.25      0.75      0.4      0.5

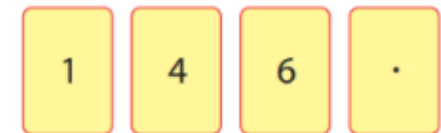
Circle the equivalent fraction to 0.25.

$\frac{2}{5}$        $\frac{5}{2}$        $\frac{25}{100}$        $\frac{100}{25}$

Round to the nearest whole number.

$8\frac{3}{8}$       8.38      8.83

Using these cards can you make a number between 4.1 and 4.61?

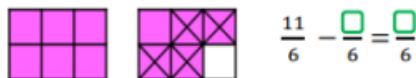


What is the smallest number you can make using all four cards?

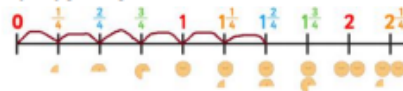
What is the largest number you can make using all four cards?

Add and subtract fractions with the same denominator including bridging over whole numbers e.g.  $\frac{7}{9} + \frac{4}{9} = \frac{11}{9}$  or 1 whole and  $\frac{2}{9}$

As before, use cubes and numicon to create the fractions:



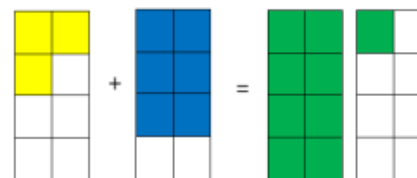
Count up and down in fraction amounts on a number line.



*Twinkl*

Count up in fraction amounts using paper cards.

*I See*



Make sure the numerators are the same, then add the denominators.

If your answer is an improper fraction, convert it to a mixed number if the problem requires it.

e.g.  $\frac{7}{9} + \frac{4}{9} = \frac{11}{9}$  or  $1\frac{2}{9}$

## Year 5 Fractions

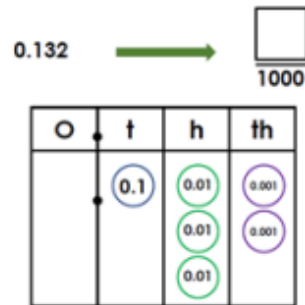
Pupils should be taught to:

- compare and order fractions whose denominators are all multiples of the same numbers
- identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements  $> 1$  as a mixed number [for example,  $5 \frac{2}{6} + 5 \frac{4}{6} = 5 \frac{6}{6} = 1 \frac{5}{6}$ ]
- add and subtract fractions with the same denominator and denominators that are multiples of the same number
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams read and write decimal numbers as fractions [for example,  $0.71 = \frac{71}{100}$ ]
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with two decimal places to the nearest whole number and to one decimal place
- read, write, order and compare numbers with up to three decimal places
- solve problems involving number up to three decimal places
- recognise the percent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator

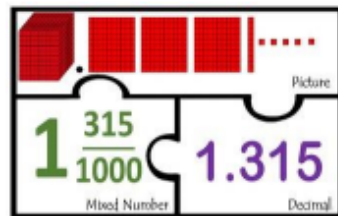
100, and as a decimal

- solve problems which require knowing percentage and decimal equivalents of  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{2}{5}$ ,  $\frac{4}{5}$  and those fractions with a denominator of a multiple of 10 or 25.

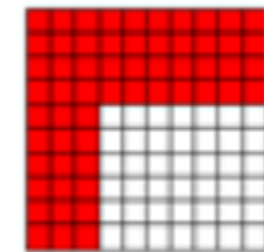
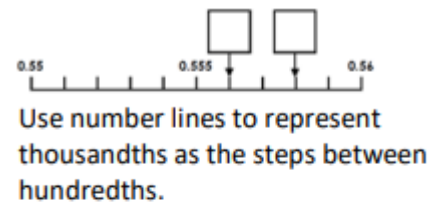
Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents



Use place value counters and grid to represent decimal numbers up to three decimal points and convert to fractions



Using base ten to physically represent decimal numbers.



Using a tens frame, 100 square, or thousands grid to represent tenths, hundredths and thousandths

**Task** Read this number: 4.325. Decide if any student showed 4.325 correctly. Explain your thinking. Then show 4.325 in another way. The large cube = 1.

Joslyn: 4 wholes, 3 tenths, 25 thousandths  
 Ryan: 3 wholes, 10 tenths, 2 hundredths, 5 thousandths  
 Jasmin: 3 wholes, 13 tenths, 1 hundredth, 15 thousandths

One tenth of £41  
 One hundredth of £41  
 One thousandth of £41  
 Continue the pattern  
 What do you notice?

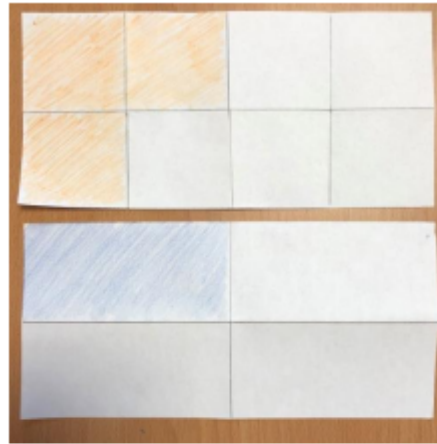
$0.085 + 0.015 = 0.1$   
 $0.075 + 0.025 = 0.1$   
 $0.065 + 0.035 = 0.1$   
 Continue the pattern for the next five number sentences.

One thousandth of my money is 31p.  
 How much do I have?

True or false?  
 0.1 of a kilometre is 1m.  
 0.2 of 2 kilometres is 2m.  
 0.3 of 3 kilometres is 3m  
 0.25 of 3m is 500cm.  
 $\frac{2}{5}$  of £2 is 20p

True or false?  
 25% of 23km is longer than 0.2 of 20km.  
 Convince me.

Compare and order fractions whose denominators are all multiples of the same number



Fraction tiles can be laid out to compare different sized fractions.



or



For example:

Order the following fractions from smallest to largest:

$$\frac{2}{3}, \frac{3}{4}, \frac{2}{6}, \frac{1}{12}, \frac{5}{6}$$

Which is larger;

$$\frac{5}{8} \text{ or } \frac{3}{4}$$

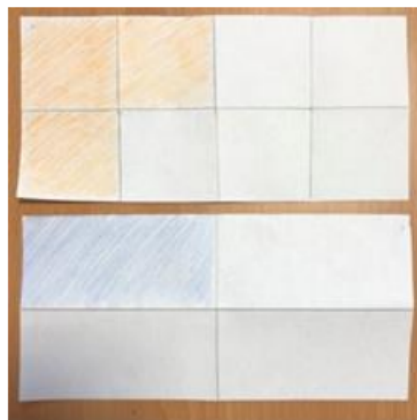
Using the number cards, can you complete the equation. Can you find more than one possibility?

$$\frac{7}{11} < \frac{\square}{\square} < \frac{8}{11}$$

- 11 15 22 25 33

Add and subtract fractions with different denominators where one denominator is a multiple of the other within the calculation

$$\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{2}{8} =$$



Using paper strips, create fractions where one denominator is a multiple of the other and demonstrate how to convert one denominator into the other by splitting the fraction, so the denominators are equal.

$$\frac{1}{3} - \frac{1}{12} =$$



$$\frac{1}{3} + \frac{5}{6} =$$



$$\frac{2}{5} - \frac{1}{10} = \frac{4}{10} - \frac{1}{10} = \frac{3}{10}$$

## Year 6 Fractions

Pupils should be taught to:

- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions  $> 1$
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example,  $4 \frac{1}{2} \times 2 \frac{1}{2} = 8 \frac{1}{2}$ ]
- divide proper fractions by whole numbers [for example,  $3 \frac{1}{2} \div 2 = 6 \frac{1}{4}$ ]
- associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example,  $\frac{3}{8}$ ]
- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places
- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places
- solve problems which require answers to be rounded to specified degrees of accuracy



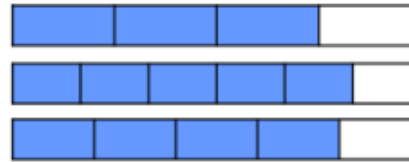
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.

Compare and order fractions, including fractions  $>1$

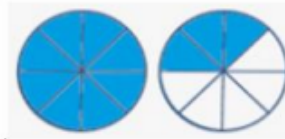


Building on to:  
Which is greater?

Order the fractions represented by the bar models in descending order:



Compare the fractions below. Explain which is larger and how you can prove it.



Order the fractions below in descending order:

$$\frac{5}{6} \quad \frac{1}{4} \quad \frac{3}{10}$$

$$\frac{2}{5} \quad \frac{1}{2}$$

Use  $<$   $>$  or  $=$  to make the statement correct:

$$1\frac{2}{7} \bigcirc 1\frac{4}{5}$$

$$\frac{4}{\square} > \frac{3}{4}$$

$$\frac{\square}{4} < \frac{2}{5}$$

$$\frac{4}{\square} = \frac{1}{2}$$

Can you use the digit cards to make the statement true using improper fractions

16 50 24 51

$$\frac{\square}{\square} < \frac{\square}{\square}$$

## Add and subtract fractions with different denominators

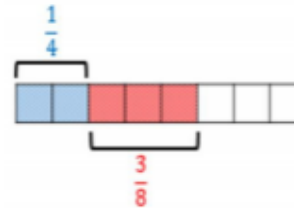
Once they have combined their previous knowledge of +/- fractions with their knowledge of equivalent fractions, the process is as above.



$$\frac{1}{2} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{5}{8}$$



$$\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}$$



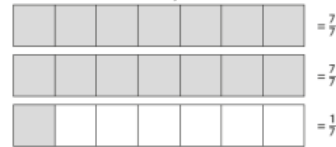
$$\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20} \text{ or } 1\frac{3}{20}$$

$$\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20} \text{ or } 1\frac{3}{20}$$

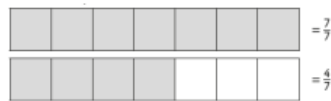
Add and subtract fractions, including mixed numbers and improper fractions where the no bridging is required

**Addition**

Provide pupils with strips split into sevenths. Elicit that  $\frac{15}{7}$  is the same  $2\frac{1}{7}$



Elicit that  $\frac{11}{7}$  is the same  $1\frac{4}{7}$



Question: How many sevenths do we have altogether? Twenty six sevenths.

Reminder: when adding/subtracting two fractions, the denominator stays the same – even when it is an improper fraction.

Elicit that  $\frac{26}{7}$  is equivalent to  $3\frac{5}{7}$

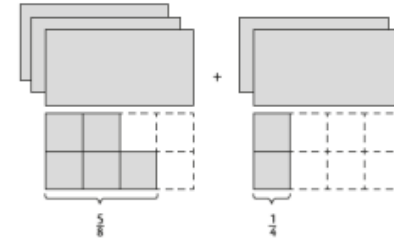
Use a similar example to model the same for subtraction.

Oscar was running a race. He ran  $3\frac{3}{8}$  of a kilometre in the first 15 minutes.

He ran  $2\frac{1}{4}$  of a kilometre in the second 15 minutes.

How far had he run in total after 30 minutes?

Share the following image:



Elicit that one full sheet of paper represents 1km. Agree that he has run five full kilometres because there are five full sheets.

Ask: Oscar ran further than 5 km. How much further? Agree that Oscar has also ran  $\frac{5}{8}$  km and  $\frac{1}{4}$  km. Establish that these are related fractions because one denominator is a multiple of the other.

Share the following then get pupils to discuss what is happening:

what is happening:

$$\begin{aligned} 3\frac{5}{8} + 2\frac{1}{4} &= 3\frac{5}{8} + 2\frac{2}{8} \\ &= (3 + \frac{5}{8}) + (2 + \frac{2}{8}) \\ &= (3 + 2) + (\frac{5}{8} + \frac{2}{8}) \\ &= (5 + \frac{7}{8}) \\ &= 5\frac{7}{8} \end{aligned}$$

*Collins Shanghai Y6 Unit 6.7*

Use a similar example to model the same for subtraction.

$$3\frac{3}{4} + \frac{1}{5} = 3\frac{19}{20}$$

$$11\frac{3}{4} - 7\frac{1}{6} = 4\frac{7}{12}$$

Add and subtract fractions, including mixed numbers and improper fractions where bridging is required

**Addition**

Example:  $3\frac{5}{7} + 2\frac{4}{7}$

Use the same method as previously with the strips, but get the children to cut up the fraction elements into the component sevenths.

This would leave 5 wholes (5 strips of  $\frac{7}{7}$ ) and nine  $\frac{1}{7}$  parts. Elicit that nine  $\frac{1}{7}$  parts is the same as  $\frac{9}{7}$  or  $1\frac{2}{7}$  parts. Combine 5 with  $1\frac{2}{7}$  to get the answer of  $6\frac{2}{7}$

**Subtraction**

Example:  $4\frac{2}{7} - 2\frac{5}{7}$

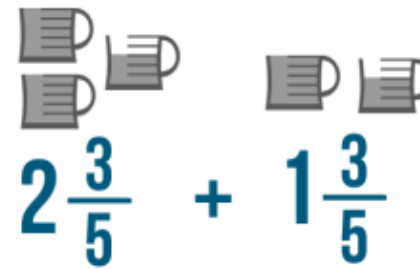
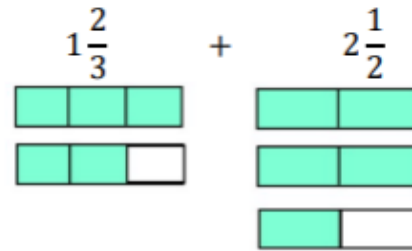
Use the same method as previously with the strips, but get the children to cut up the fraction elements into the component sevenths.

This would leave 2 wholes (2 strips of  $\frac{7}{7}$ ) but then they would realise that they cannot take  $\frac{5}{7}$  away from  $\frac{2}{7}$ .

Discuss what could be done next.

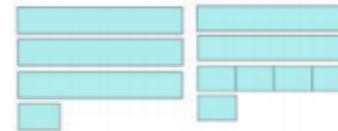
Elicit that they could cut one of their two wholes into  $\frac{7}{7}$ . Going back to their original  $4\frac{2}{7}$  they would re-partition that into  $3\frac{9}{7}$ .

With  $3\frac{9}{7}$  in strips, they could remove the 2 wholes then the  $\frac{4}{7}$  leaving them with  $1\frac{5}{7}$ .



Calculate  $3\frac{1}{4} - 1\frac{3}{4}$

$3\frac{1}{4}$  can become  $2\frac{5}{4}$



For example

$3\frac{3}{4} + \frac{7}{5} = 5\frac{3}{20}$

$11\frac{3}{4} - 7\frac{5}{6} = 3\frac{11}{12}$

Multiply simple pairs of proper fractions, writing the answer in its simplest form Multiply fractions by whole numbers



Use Cuisenaire Rods and fraction towers to demonstrate multiplying fractions by an integer.

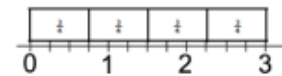


Using fraction towers to demonstrate the abstract:  
 '3 lots of  $\frac{1}{4}$ '  
 2 lots of  $\frac{2}{6}$  =

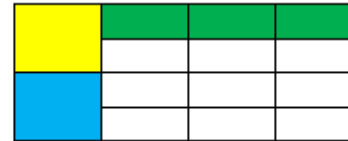
Use bar models to work out  $3 \times \frac{1}{4}$  =



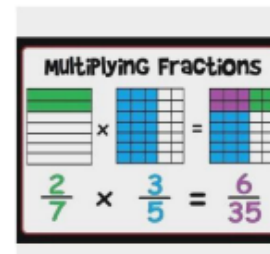
Use a number line to work it out:



Use a diagram to represent multiplying fractions. Build an array (as used when multiplying whole numbers) For example  $\frac{1}{4} \times \frac{2}{4}$



Draw a bar and shade  $\frac{1}{4}$   
 Draw an adjoining column and shade  $\frac{2}{4}$ .  
 The shaded cells represent the total. ( $\frac{2}{16}$  or  $\frac{1}{8}$ )



Solve:

$$\frac{2}{x} \times \frac{3}{5} = \frac{6}{20} = \frac{3}{10}$$

$$\frac{2}{5} \times \frac{3}{6} = \frac{10}{60} = \frac{1}{6}$$

How many ways can you answer the following?

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

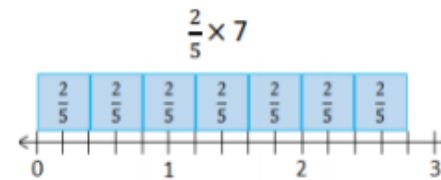
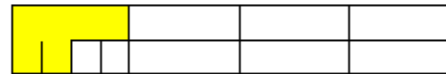
In each number sentence, replace the boxes with different whole numbers less than 20 so that the number sentence is true.

Draw your own bar model to work out the following:

$$2 \times \frac{2}{3} = \quad 4 \times \frac{2}{3} =$$



Find  $\frac{2}{4}$  of  $\frac{1}{4}$  ( $\frac{2}{4} \times \frac{1}{4}$ )



$$\frac{1}{\square} \times \frac{3}{\square} = \frac{\square}{\square}$$

$$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{8}{15}$$

$$\frac{2}{\square} \times \frac{5}{\square} < \frac{10}{\square}$$

$$\frac{\square}{\square} \div 3 = \frac{1}{\square}$$

$$\frac{\square}{\square} \div 3 > \frac{1}{4}$$

Divide proper fraction by whole numbers.

Using paper strips is a good way to demonstrate what happens to a fraction when it is divided by a whole integer i.e. 2

Fold the paper in half:

**What happens when I divide  $\frac{1}{2}$  by 2?**

By folding each half into two parts, it becomes clear that quarters have been formed.

Lee has  $\frac{2}{5}$  of a chocolate bar. He shares it with his friend.  
How much chocolate do they get each?

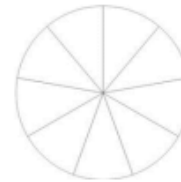


Use the diagrams to help you calculate:



Calculate the following and use the diagram to help you.

$$\frac{1}{8} \div 4 =$$



Solve:

$$6 \div \quad = 9$$

$$\quad \div \frac{2}{5} = 10$$

$$\quad \div \quad = 5\frac{5}{6}$$

Harjoht's Mum ordered pizza for the whole family. Harjoht ate  $\frac{1}{4}$  of the pizza. His Mum, brother and sister ate the rest of the pizza. What fraction of the pizza did they get each?

Molly's Mum ordered pizza for the whole family. Molly ate  $\frac{1}{4}$  of the pizza. Six members of the family eat the remaining pizza. What fraction of the pizza did they get each?